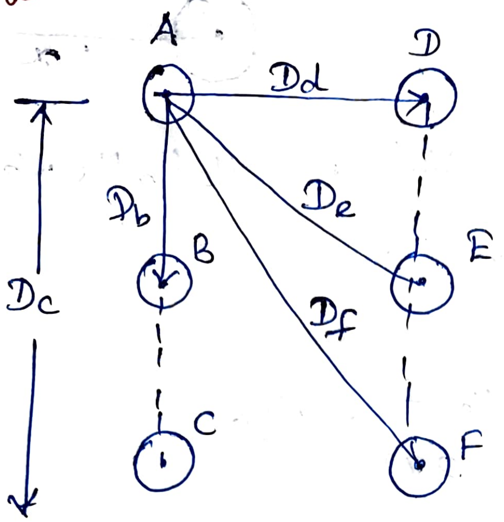
Flux linkage of parallel current carrying conductors.



Consider 6 conductors A, B, C, D, E & F carrying currents I_A, I_B, I_C, I_D, I_E & I_F respectively.

D_b, D_c, D_d, D_e & D_f are the distances between the conductors.

Magnetic field & flux will be created inside & outside the conductors.

Let us find the total flux linkage with conductor A.

Flux linkage with conductor A due to its own current $= \mu_0 \frac{I_A}{2\pi}\left[\frac{1}{4} + \int_r^{\infty} \frac{dx}{x}\right]$

Flux linkage ~~per~~ with A due to current $I_B = \mu_0 \frac{I_B}{2\pi} \int_{D_b}^{\infty} \frac{dx}{x}$

Flux linkage with A due to current I_c $= \mu_0 \dfrac{I_c}{2\pi} \displaystyle\int_{D_C}^{\infty} \dfrac{dx}{x}$.

$$I_D = \mu_0 \frac{I_D}{2\pi} \int_{D_d}^{\infty} \frac{dx}{x}$$

$$I_E = \mu_0 \frac{I_E}{2\pi} \int_{D_e}^{\infty} \frac{dx}{x}$$

$$I_f = \mu_0 \frac{I_F}{2\pi} \int_{D_f}^{\infty} \frac{dx}{x}$$

Total flux linkage with the conductor A :

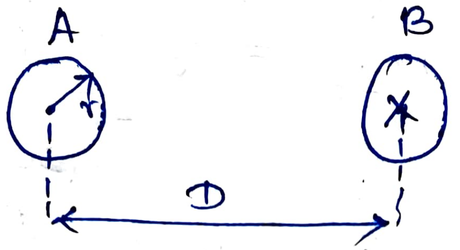
$$\psi_A = \mu_0 \frac{I_A}{2\pi}\left[\frac{1}{4} \int_{r}^{\infty} \frac{dx}{x}\right] + \mu_0 \frac{I_B}{2\pi} \int_{D_b}^{\infty} \frac{dx}{x} +$$

$$\mu_0 \frac{I_c}{2\pi} \int_{D_c}^{\infty} \frac{dx}{x} + \mu_0 \frac{I_D}{2\pi} \int_{D_d}^{\infty} \frac{dx}{x} + \mu_0 \frac{I_E}{2\pi} \int_{D_e}^{\infty} \frac{dx}{x}$$

$$+ \mu_0 \frac{I_F}{2\pi} \int_{D_f}^{\infty} \frac{dx}{x}.$$ //

Inductance of a single phase Tr. line.

Let us consider a single phase line which consists of two parallel conductors A and B of radius r metres and spaced D metres apart as shown in fig.



Conductors A & B are carrying currents I_A & I_B. Both are same in magnitude but in opposite direction, hence one conductor acts as a return path for the other conductor.

$$I_A = -I_B.$$

Let us find the total flux linkage with conductor A.

flux linkage with A $= \mu_0 \frac{I_A}{2\pi}\left[\frac{1}{4}\int_r^\infty \frac{dx}{x}\right]$

External flux. $I_B = \mu_0 \frac{I_B}{2\pi}\int_D^\infty \frac{dx}{x}$

Total flux linkage with conductor A

$$\psi_A = \mu_0 \frac{I_A}{2\pi}\left[\frac{1}{4} + \int_r^\infty \frac{dx}{x}\right] + \mu_0 \frac{I_B}{2\pi}\int_D^\infty \frac{dx}{x}.$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A\left[\ln x\right]_r^\infty + I_B\left[\ln x\right]_D^\infty\right]$$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + I_A \ln\infty - I_A \ln r + I_B \ln\infty - I_B \ln D\right].$$

$\because I_A = -I_B.$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \ln\infty\,(I_A + I_B) - I_A \ln r + I_A \ln D\right]$$

$\because I_A + I_B = 0.$

$$= \frac{\mu_0}{2\pi}\left[\frac{I_A}{4} + \ln\infty\,(0) + I_A \ln\frac{D}{r}\right]$$

$$\psi_A = \frac{\mu_0 I_A}{2\pi}\left[\frac{1}{4} + \ln\frac{D}{r}\right] \text{ wb-turn / meter length}$$

Inductance of conductor A.

$$L_A = \frac{\psi_A}{I_A} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{D}{r}\right] \text{ H/m.}$$

Sub $\mu_0 = 4\pi \times 10^{-7}$

$$L_A = 2\times10^{-7}\left[\frac{1}{4} + \ln\frac{D}{r}\right]$$

$$= 2\times10^{-7}\left[\ln e^{1/4} + \ln\frac{D}{r}\right]$$

$$= 2\times10^{-7}\ln\left[\frac{D}{re^{-1/4}}\right]$$

$$\boxed{L_A = 2\times10^{-7}\ln\frac{D}{r'}}$$

where $r' = 0.7788\,r$ Geometric Mean Radius (GMR)

|||ly for conductor B,

$$L_B = \frac{\psi_B}{I_B} = \frac{\mu_0}{2\pi}\left[\frac{1}{4} + \ln\frac{D}{r}\right], \ H/m$$

Loop inductance of 1φ line is

$$2L_A = 4\times10^{-7}\left[\frac{1}{4} + \ln\frac{D}{r}\right] H/m$$

$$\boxed{= 10^{-7}\left[1 + 4\ln\frac{D}{r}\right] H/m}$$

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