SNS COLLEGE OF ENGINEERING

Kurumbapalayam(Po), Coimbatore – 641 107
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Department of Information Technology

Course Name – COMPUTER GRAPHICS

III Year / V Semester

Unit 1 – INTRODUCTION TO COMPUTER GRAPHICS

Topic : Parametric Form for a Curve

PARAMETRIC CURVES

- Curves having parametric form are called parametric curves. The explicit and implicit curve representations can be used only when the function is known. In practice the parametric curves are used. A two-dimensional parametric curve has the following form −

$$P(t) = f(t), g(t) \text{ or } P(t) = x(t), y(t)$$

- The functions f and g become the (x, y) coordinates of any point on the curve, and the points are obtained when the parameter t is varied over certain interval [a, b], normally [0, 1].

BEZIER CURVES

Bezier curve is discovered by the French engineer Pierre Bézier. These curves can be generated under the control of other points. Approximate tangents by using control points are used to generate curve. The Bezier curve can be represented mathematically as

$$\sum_{k=0}^{n} P_i B_i^n(t)$$

Where p_i

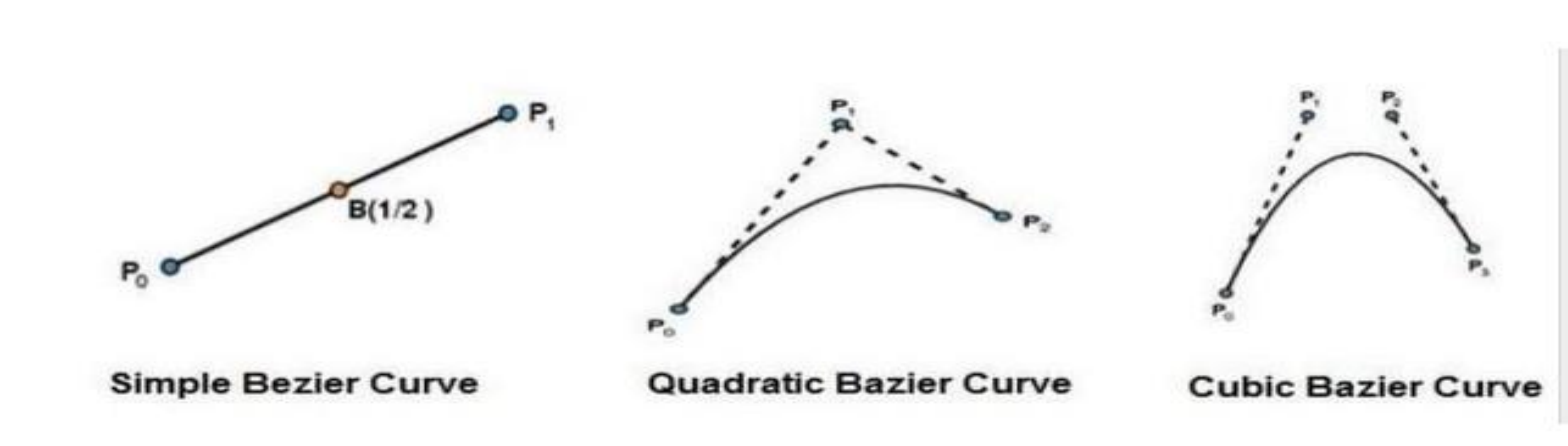
is the set of points and $B_i^n(t)$

represents the Bernstein polynomials which are given by −

$$B_i^n(t) = \binom{n}{i}(1-t)^{n-i}t^i$$

BEZIER CURVES

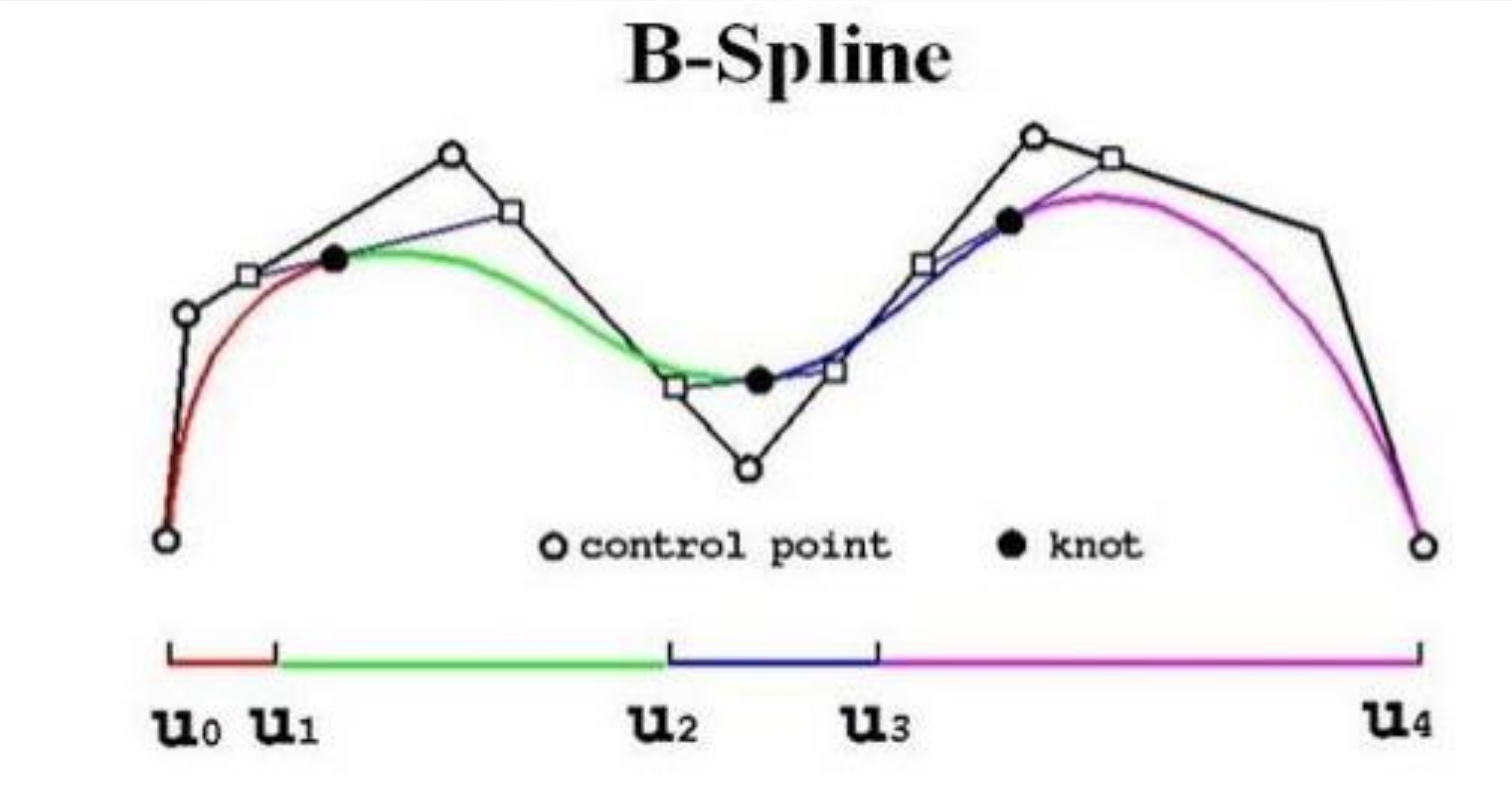
- Where n is the polynomial degree, i is the index, and t is the variable.
- The simplest Bézier curve is the straight line from the point P0 to P1 .
- A quadratic Bezier curve is determined by three control points. A cubic Bezier curve is determined by four control points.



B-SPLINE CURVES

- The Bezier-curve produced by the Bernstein basis function has limited flexibility.
    - First, the number of specified polygon vertices fixes the order of the resulting polynomial which defines the curve.
    - The second limiting characteristic is that the value of the blending function is nonzero for all parameter values over the entire curve.
- The B-spline basis contains the Bernstein basis as the special case. The B-spline basis is non global.

B-SPLINE CURVES



B-SPLINE CURVES

A B-spline curve is defined as a linear combination of control points Pi and B-spline

basis function Ni,

k (t) given by

$$C(t)=\sum_{ni=0} P_i N_{i,k}(t),$$

$$n \geq k-1,\ t \epsilon [tk-1, tn+1]$$

Where,

- {pi : i=0, 1, 2….n} are the control point

B-SPLINE CURVES

- k is the order of the polynomial segments of the B-spline curve. Order k means that the curve is made up of piecewise polynomial segments of degree k - 1,
- the $N_{i,k}(t)$
  - are the “normalized B-spline blending functions”. They are described by the order k and by a non-decreasing sequence of real numbers normally called the “knot sequence”.

$$t_i : i = 0, \ldots n+K$$

The N_i, k functions are described as follows −

$$N_{i,1}(t) = \{1, 0, if u \epsilon [t_i, t_{i+1}) Otherwise$$

and if k > 1,

$$N_{i,k}(t) = t - t_i t_{i+k-1} N_{i,k-1}(t) + t_{i+k} - t t_{i+k} - t_{i+1} N_{i+1,k-1}(t)$$

and

$$t \epsilon [t_{k-1}, t_{n+1})$$

Comparison of Bezier Curve and B-Spline/Curve:

Sr. No.	Bezier Curve	B-Spline Curve
1.	They are not designed with sharp bend and cornered curves.	They are designed with sharp bend and even corners.
2.	A change in a portion of a curve causes the whole of the curve to change.	B-Spline has local over the curve. So, whole of the curve need not change.
3.	They are special case of B-Spline curves.	They are not special case of Bezier curves.
4.	They are less flexible.	They are more flexible with large number of control points.
5.	They interpolate their first and last control points.	They do not interpolate their first and last control points.

THANK YOU