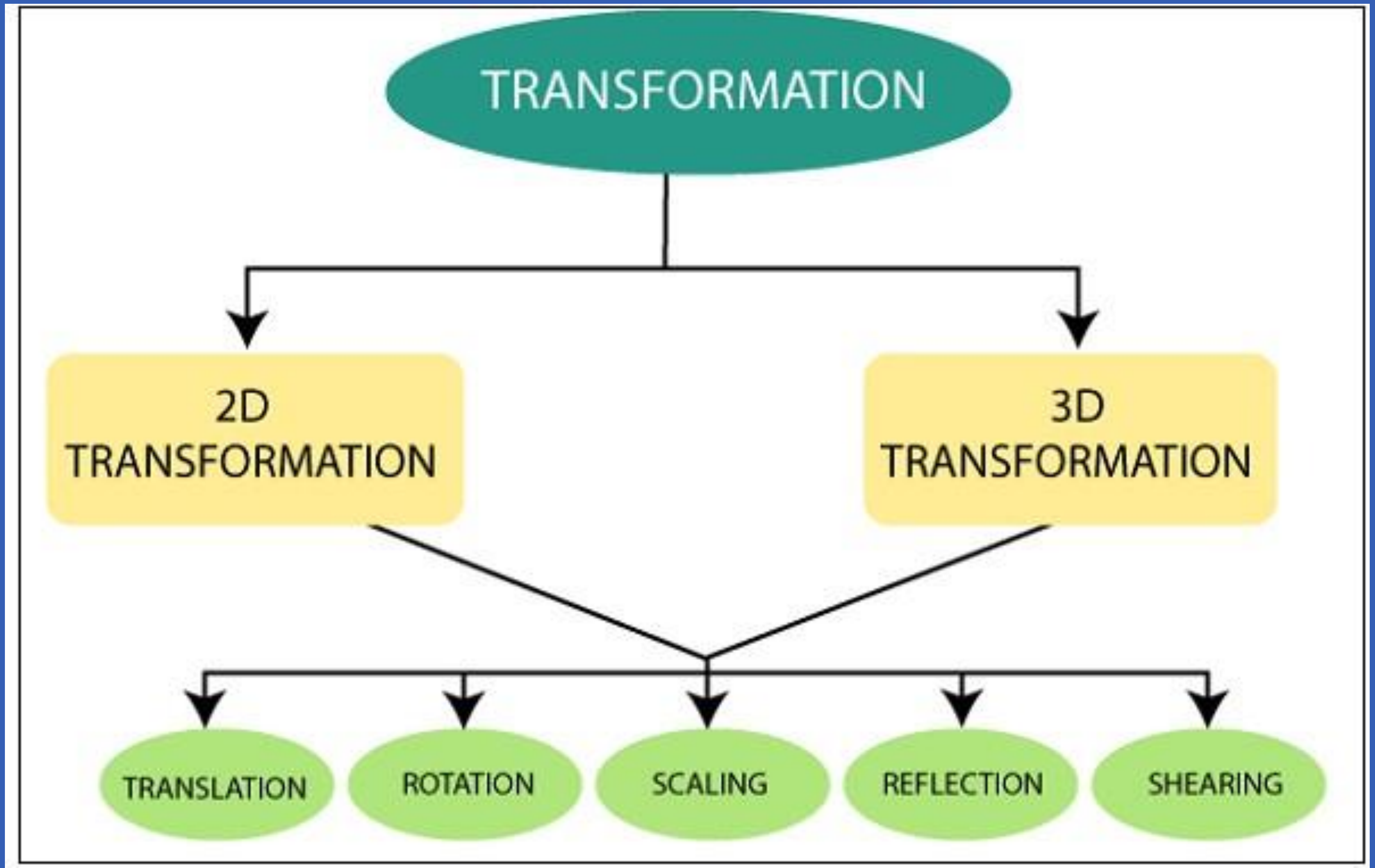
INTRODUCTION TO

TRANSFORMATION

Transformation

- In computer graphics, transformation refers to the process of changing the position, size, or orientation of an object.
- It is used to manipulate and animate objects in a virtual environment.



TRANSLATION

- Translation is a process of moving an object from one position to another in a two dimensional plane.

- To translate a point from P (x_0, y_0) to Q (x_1, y_1), then we have to add Translation coordinates (Tx, Ty) with original coordinates.

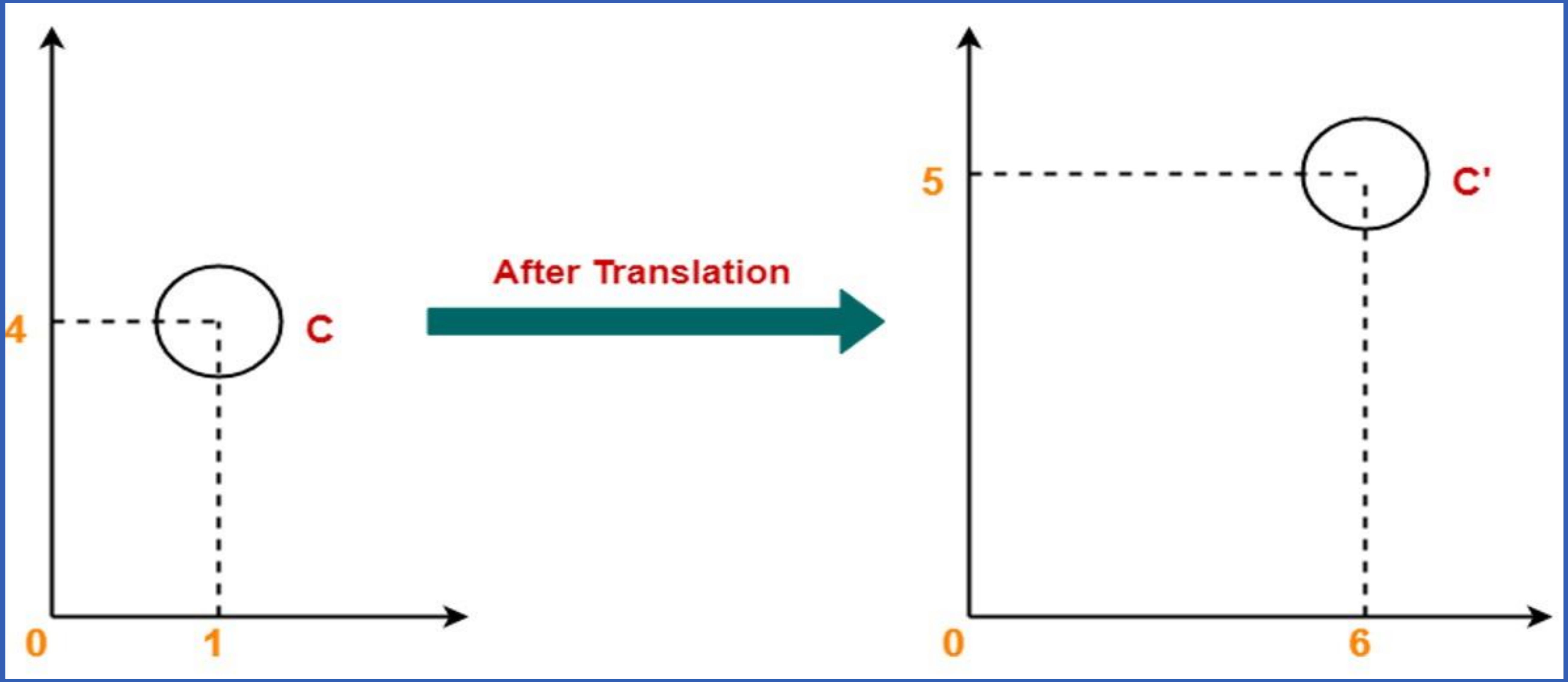
Translation equation

$$x1 = x + Tx$$

$$y1 = y + Ty$$

(The translation pair (Tx, Ty) is called as shift vector)

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$



4
C
0
1
After Translation
5
C'
0
6

Problem

Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Solution-

- Old center coordinates of C = (Xold, Yold) = (1,4)
- Translation vector = (Tx, Ty) = (5, 1)

Let the new center coordinates of C = (Xnew, Ynew).

Applying the translation equations, we have-

- Xnew = Xold + Tx = 1 + 5 = 6
- Ynew = Yold + Ty = 4 + 1 = 5

Thus, New center coordinates of C = (6, 5).

In matrix form, the new center coordinates of C after translation may be obtained as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Thus, New center coordinates of C = (6, 5).

ROTATION

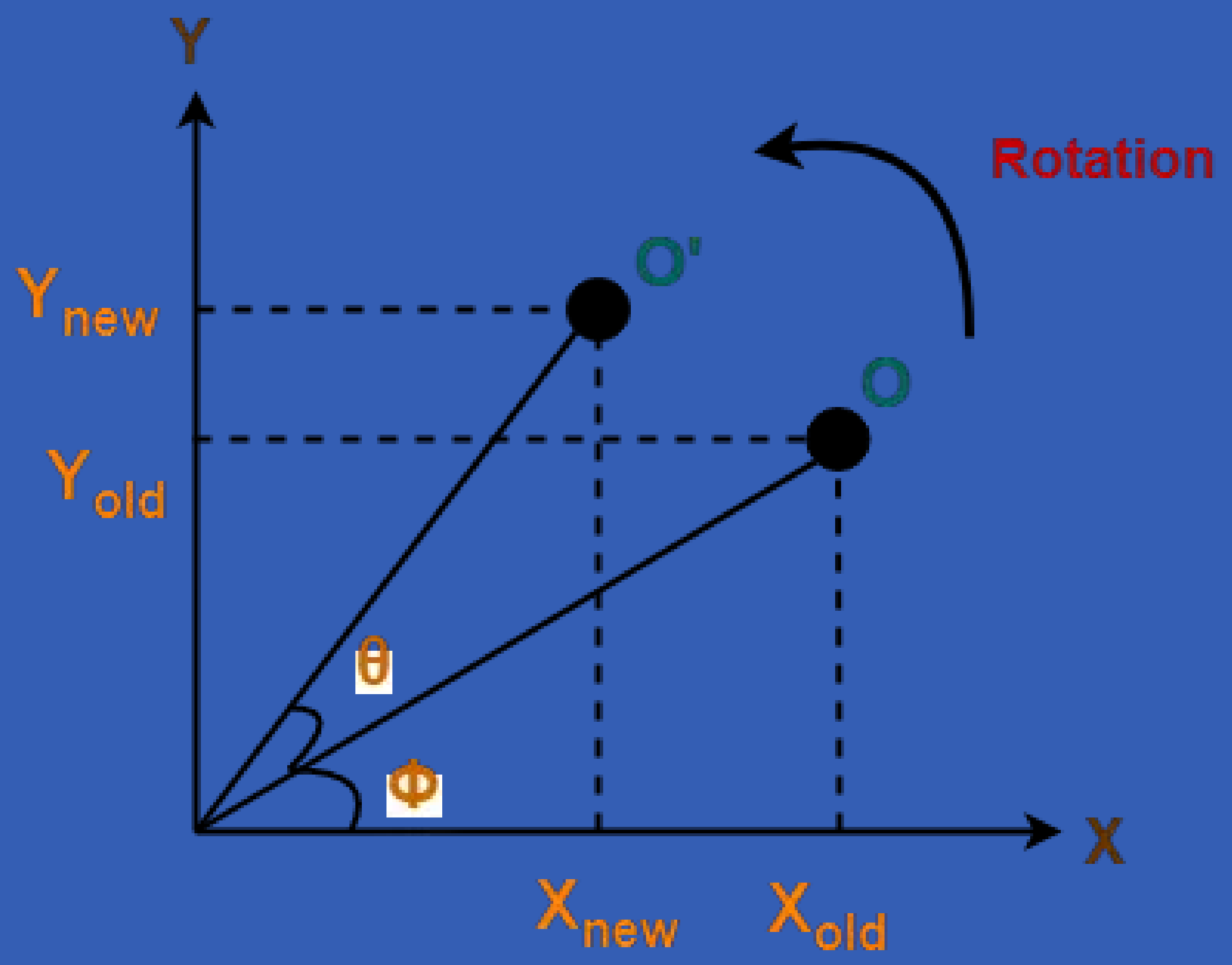
- 2D Rotation is a process of rotating an object with respect to an angle in a twodimensional plane.
- Let

Initial coordinates of the object O = (Xold, Yold)

Initial angle of the object O with respect to origin = Φ

Rotation angle = θ

New coordinates of the object O after rotation = (Xnew, Ynew)



This rotation is achieved by using the following rotation equations-

$X_{new} = X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$Y_{new} = X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

In Matrix form, the above rotation equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Problem:

Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line

Solution :

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

Old ending coordinates of the line = (X_{old}, Y_{old}) = (4, 4)

Rotation angle = θ = 30º

Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

Xnew

$= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$

$= 4 \times \cos 30^\circ - 4 \times \sin 30^\circ$

$= 4 \times (\sqrt{3}/2) - 4 \times (1/2)$

$= 2\sqrt{3} - 2$

$= 2(\sqrt{3} - 1)$

$= 2(1.73 - 1)$

$= 1.46$

Ynew

$= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$

$= 4 \times \sin 30^\circ + 4 \times \cos 30^\circ$

$= 4 \times (1/2) + 4 \times (\sqrt{3}/2)$

$= 2 + 2\sqrt{3}$

$= 2(1 + \sqrt{3})$

$= 2(1 + 1.73)$

$= 5.46$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).

In matrix form, the new ending coordinates of the line after rotation may be obtained as-

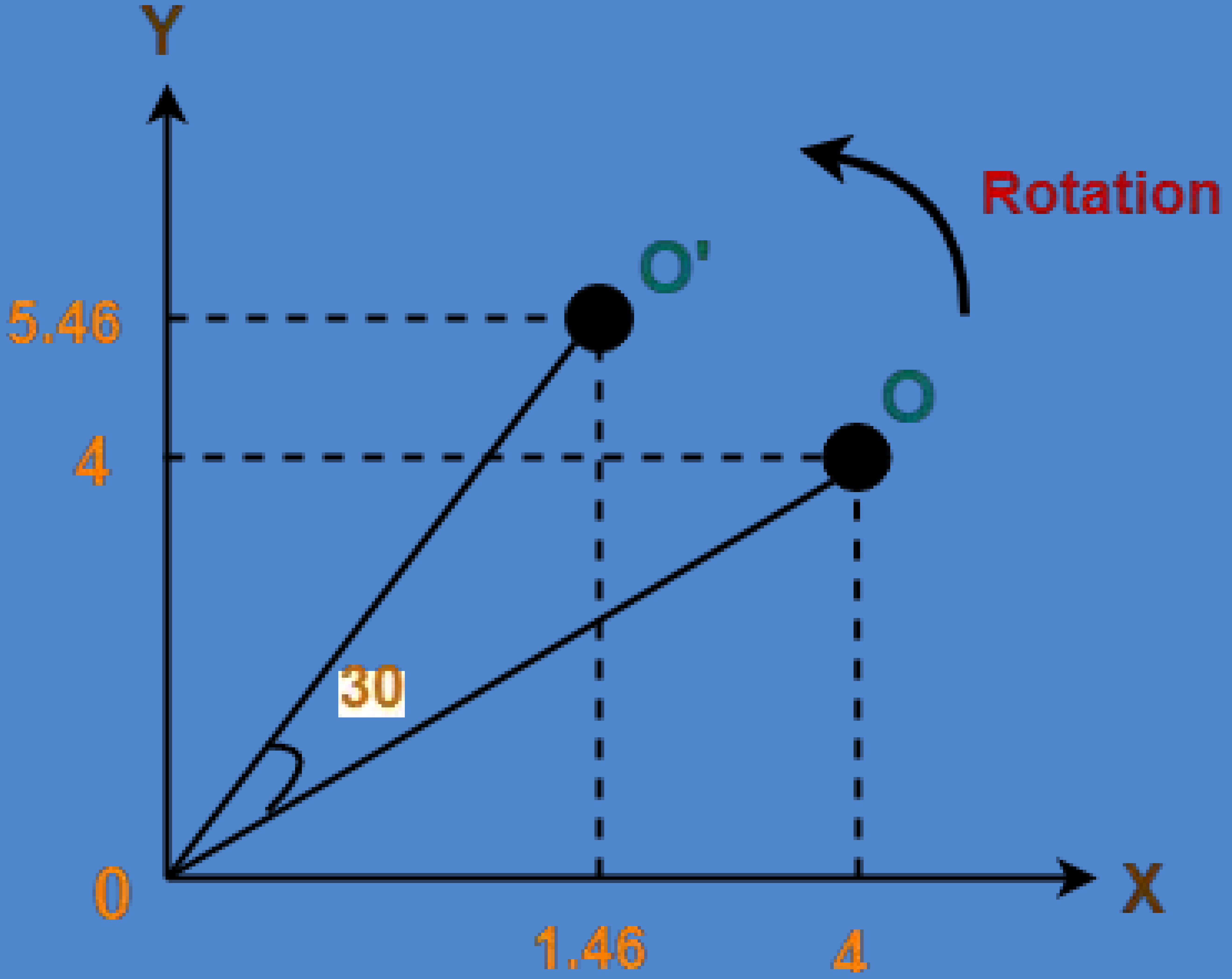
$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$


Y
Rotation
O'
5.46
O
4
30
0
1.46
4
X

SCALING

- scaling is a process of modifying or altering the size of objects.
- Scaling may be used to increase or reduce the size of object.
- If scaling factor > 1, then the object size is increased.
- If scaling factor < 1, then the object size is reduced.

This scaling is achieved by using the following scaling equations-

- Xnew = Xold x Sx
- Ynew = Yold x Sy

Initial coordinates of the object O = (Xold, Yold)
Scaling factor for X-axis = Sx
Scaling factor for Y-axis = Sy
New coordinates of the object O after scaling = (Xnew, Ynew)

In Matrix form, the above scaling equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Scaling Matrix

Problem:

Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object

Solution-

Given-

Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)

Scaling factor along X axis = 2

Scaling factor along Y axis = 3

For Coordinates A(0, 3)

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9).

For Coordinates B(3, 3)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$

$Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 3 \times 2 = 6$

$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

For Coordinates D(0, 0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

$X_{new} = X_{old} \times S_x = 0 \times 2 = 0$

$Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).