

Line Drawing Algorithm

- The Line drawing algorithm is a graphical algorithm which is used to represent the line segment on discrete graphical media, i.e., printer and pixel-based media.
- A line contains two points. The point is an important element of a line.

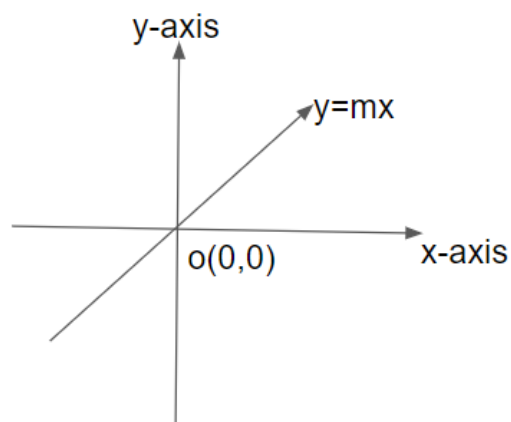
properties of a Line Drawing Algorithm

- **An algorithm should be precise:** Each step of the algorithm must be adequately defined.
- **Finiteness:** An algorithm must contain finiteness. It means the algorithm stops after the execution of all steps.
- **Easy to understand:** An algorithm must help learners to understand the solution in a more natural way.
- **Correctness:** An algorithm must be in the correct manner.
- **Effectiveness:** The steps of an algorithm must be valid and efficient.
- **Uniqueness:** All steps of an algorithm should be clearly and uniquely defined, and the result should be based on the given input.
- **Input:** A good algorithm must accept at least one or more input.
- **Output:** An algorithm must generate at least one output

The formula for a slope line interception is:

$$Y = mx + b$$

In this formula, m is the slope line and b is the line's intercept of y . Two endpoints for the line segment are supplied in coordinates (x_1, y_1) and (x_2, y_2) .



Types of Line Drawing Algorithm

- DDA (Digital Differential Analyzer) Line Drawing Algorithm
- Bresenham's Line Drawing Algorithm

DDA (Digital Differential Analyzer)

Digital Differential Analyzer algorithm is also known as an incremental method of scan conversion.

Algorithm of Digital Differential Analyzer (DDA) Line Drawing

Step 1: Start.

Step 2: We consider Starting point as (x_1, y_1) , and ending point (x_2, y_2) .

Step 3: Now, we have to calculate Δx and Δy .

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$m = \Delta y / \Delta x$$

Step 4: Now, we calculate three cases.

Case 1: If $m < 1$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k + m$$

Case 2: If $m > 1$

$$y_{k+1} = y_k + 1$$

$$x_{k+1} = x_k + 1/m$$

Case 3: If $m = 1$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k + 1$$

Step 5: We will repeat step 4 until we find the ending point of the line.

Step 6: Stop

Example: A line has a starting point $(1,7)$ and ending point $(11,17)$. Apply the Digital Differential Analyzer algorithm to plot a line.

Step 1: Consider Starting Point = $(x_1, y_1) = (1,7)$
Ending Point = $(x_2, y_2) = (11,17)$

Step 2: calculate m

$$\Delta x = x_2 - x_1 = 11 - 1 = 10$$

$$\Delta y = y_2 - y_1 = 17 - 7 = 10$$

$$m = \Delta y / \Delta x = 10/10 = 1$$

Step 3: We get $m = 1$, Third case is satisfied

x_k	y_k	x_{k+1}	y_{k+1}	(x_{k+1}, y_{k+1})
1	7	2	8	(2, 8)
		3	9	(3, 9)
		4	10	(4, 10)
		5	11	(5, 11)
		6	12	(6, 12)
		7	13	(7, 13)
		8	14	(8, 14)
		9	15	(9, 15)
		10	16	(10, 16)
		11	17	(11, 17)

Step 4: We will repeat step 3 until we get the endpoints of the line.

Step 5: Stop.

The coordinates of drawn line are

$$P_1 = (2, 8)$$

$$P_2 = (3, 9)$$

$$P_3 = (4, 10)$$

$$P_4 = (5, 11)$$

$$P_5 = (6, 12)$$

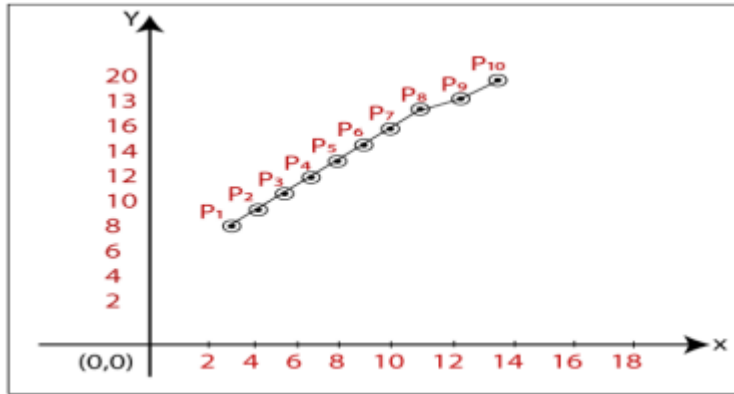
$$P_6 = (7, 13)$$

$$P_7 = (8, 14)$$

$$P_8 = (9, 15)$$

$$P_9 = (10, 16)$$

$$P_{10} = (11, 17)$$



Bresenham's Line Drawing Algorithm

- This algorithm was introduced by "Jack Elton Bresenham" in 1962.
- This algorithm helps us to perform scan conversion of a line.
- It is a powerful, useful, and accurate method.
- We use incremental integer calculations to draw a line. The integer calculations include addition, subtraction, and multiplication.

Algorithm of Bresenham's Line Drawing Algorithm

Step 1: Start.

Step 2: Now, we consider Starting point as (x_1, y_1) and ending point (x_2, y_2) .

Step 3: Now, we have to calculate Δx and Δy .

$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$m = \Delta y / \Delta x$$

Step 4: Now, we will calculate the decision parameter p_k with following formula.

$$p_k = 2\Delta y - \Delta x$$

Step 5: The initial coordinates of the line are (x_k, y_k) , and the next coordinates are (x_{k+1}, y_{k+1}) .

Now, we are going to calculate two cases for decision parameter p_k

Case 1: If $p_k < 0$ Then

$$p_{k+1} = p_k + 2\Delta y$$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k$$

Case 2: If $p_k \geq 0$ Then

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x$$

$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k + 1$$

Step 6: We will repeat step 5 until we found the ending point of the line and the total number of iterations = $\Delta x - 1$.

Step 7: Stop

Example: A line has a starting point (9,18) and ending point (14,22). Apply the Bresenham's Line Drawing algorithm to plot a line.

Step 1: Consider, Starting Point = $(x_1, y_1) = (9, 18)$

Ending Point = $(x_2, y_2) = (14, 22)$

Step 2: First, we calculate Δx , Δy .

$$\Delta x = x_2 - x_1 = 14 - 9 = 5$$

$$\Delta y = y_2 - y_1 = 22 - 18 = 4$$

Step 3: Calculate the decision parameter

$$(p_k) p_k = 2 \Delta y - \Delta x = 2 \times 4 - 5 = 3$$

The value of $p_k = 3$

Step 3: Now, we will check both the cases.

If $p_k \geq 0$ Then Case 2 is satisfied.

$$\text{Thus } p_{k+1} = p_k + 2 \Delta y - \Delta x = 3 + (2 \times 4) - (2 \times 5) = 1$$

$$x_{k+1} = x_k + 1 = 9 + 1 = 10$$

$$y_{k+1} = y_k + 1 = 18 + 1 = 19$$

Step 4: Now move to next step. We will calculate the coordinates until we reach the end point of the line.

Step 5: Stop

p_k	p_{k+1}	x_{k+1}	y_{k+1}
		9	18
3	1	10	19
1	-1	11	20
-1	7	12	20
7	5	13	21
5	3	14	22

The Coordinates of drawn lines are

$$P_1 = (9, 18)$$

$$P_2 = (10, 19)$$

$$P_3 = (11, 20)$$

$$P_4 = (12, 20)$$

$$P_5 = (13, 21)$$

$$P_6 = (14, 22)$$

