



$$L[tf(t)] = -\varphi'(s)$$

Corollary :- If $L[f(t)] = \varphi(s)$ then $L[t^n f(t)] = (-1)^n \varphi^n(s)$.

Proof : W.K.T. $L[tf(t)] = -\varphi'(s)$

$$\begin{aligned} L[t^2 f(t)] &= L[t \cdot tf(t)] \\ &= -\frac{d}{ds} L[tf(t)] \\ &= -\frac{d}{ds} \left[-\frac{d}{ds} Lf(t) \right] \\ &= (-1)^2 \frac{d^2}{ds^2} [Lf(t)] \\ &= (-1)^2 \frac{d^2}{ds^2} \varphi(s) \end{aligned}$$

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$$L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} \varphi(s) = (-1)^n \varphi^n(s)$$

PROBLEMS BASED ON TRANSFORMS OF DERIVATIVES

Example 1. Find $L[t \sin 2t]$

Solution : W.K.T. $L[t^n f(t)] = (-1)^n \varphi^n(s)$

$$\begin{aligned} L(t \sin 2t) &= -\frac{d}{ds} [L(\sin 2t)] = -\frac{d}{ds} \left[\frac{2}{s^2 + 4} \right] \\ &= - \left[\frac{-4s}{(s^2 + 4)^2} \right] = \frac{4s}{(s^2 + 4)^2} \end{aligned}$$

Example 2. Find $L[t^2 e^{-3t}]$

Solution : W.K.T $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} [\varphi(s)]$

$$\begin{aligned} L[t^2 e^{-3t}] &= (-1)^2 \frac{d^2}{ds^2} L[e^{-3t}] = \frac{d^2}{ds^2} \left[\frac{1}{s + 3} \right] \\ &= \frac{d}{ds} \left[\frac{-1}{(s + 3)^2} \right] = \frac{2}{(s + 3)^3} \end{aligned}$$



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