Solution of Linear ODE 2nd order with constant coefficients using Laplace Transformation.

$$L[y'(t)] = sL[y(t)] - y(0)$$

$$L[y''(t)] = s^2L[y(t)] - sy(0) - y'(0)$$

$$L[y'''(t)] = s^3L[y(t)] - s^2y(0) - sy(0) - y''(0)$$

(1) Solve $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2$, given $x=0$ and $\frac{dx}{dt} = 5$ for $t=0$ using Laplace transform method.

Solution

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = 2, \quad x(0)=0, \; x'(0) = 5$$

$$x''(t) - 3x'(t) + 2x(t) = 2$$

$$L[x''(t)] - 3L[x'(t)] + 2L[x(t)] = L(2)$$

$$L[x''(t)] - 3[sL(x(t)) - x(0)] + 2L[x(t)] = 2L(1)$$

$$[s^2L[x(t)] - sx(0) - x'(0)] - 3[sL(x(t))] + 2L[x(t)] = 2\left[\frac{1}{s}\right]$$

$$s^2L[x(t)] - 0 - 5 - 3[sL(x(t))]$$

$$(s^2 - 3s + 2)L[x(t)] = \frac{2}{s} + 5$$

$$L[x(t)] = \frac{2+5s}{s(s^2-3s+2)}$$

$$= \frac{2+5s}{s(s-2)(s-1)}$$

$$L[x(t)] = \frac{2+5s}{s(s-1)(s-2)} = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s-2} \rightarrow (1)$$

$$2+5s = A(s-1)(s-2) + Bs(s-2) + Cs(s-1)$$

$A = 1$, $B = -7$, $C = 6$

$$= \frac{1}{s} - 7\left(\frac{1}{s-1}\right) + 6\left(\frac{1}{s-2}\right)$$

$$x(t) = L^{-1}\left[\frac{1}{s}\right] - 7L^{-1}\left[\frac{1}{s-1}\right] + 6L^{-1}\left[\frac{1}{s-2}\right]$$

$$x(t) = 1 - 7e^{t} + 6e^{2t}$$

② Using Laplace Transform solve $y'' - 3y' + 2y = e^{-t}$ given $y(0) = 1$, $y'(0) = 0$.

solution:

Given $y'' - 3y' + 2y = e^{-t}$

i.e $L[y''(t)] - 3L[y'(t)] + 2L[y(t)] = L[e^{-t}]$

$$L[y''(t)] - 3L[y'(t)] + 2L[y(t)] = \frac{1}{s+1} \quad \text{①}$$

$$\left[s^2L[y(t)] - sy(0) - y'(0)\right] - 3\left[sL[y(t)] - y(0)\right] + 2L[y(t)] = \frac{1}{s+1}$$

Given $y(0) = 1$, $y'(0) = 0$.

① ⇒ $s^2L[y(t)] - s - 3sL[y(t)] + 3 + 2L[y(t)] = \frac{1}{s+1}$

$$(s^2 - 3s + 2)L[y(t)] = \frac{1}{s+1} + s - 3$$

$$L[y(t)] = \frac{1 + s^2 + s - 3s - 3}{s+1} = \frac{s^2 - 2s - 2}{s+1}$$

$$L[y(t)] = \frac{s^2 - 2s - 1}{(s-1)(s+1)(s-2)}$$

$A = \frac{3}{2}$, $B = \frac{1}{6}$, $C = -\frac{2}{3}$

$$L[y(t)] = \frac{3/2}{s-1} + \frac{1/6}{s+1} + \frac{(-2/3)}{s-2}$$

$$y(t) = \frac{3}{2} L^{-1}\left[\frac{1}{s-1}\right] + \frac{1}{6} L^{-1}\left[\frac{1}{s+1}\right] - \frac{2}{3} L^{-1}\left[\frac{1}{s-2}\right]$$

$$\therefore\ y(t) = \frac{3}{2} e^{t} + \frac{1}{6} e^{-t} - \frac{2}{3} e^{2t}$$