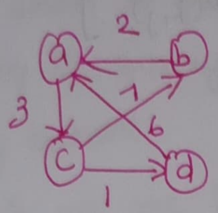
Floyd's algorithm :

This is for finding the shortest path between every pair of vertices in a given graph.

This is for both undirected and directed graph.

Also called " All pair shortest path".

Ex:



$$W = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & \infty & 3 & \infty \\ b & 2 & 0 & \infty & \infty \\ c & \infty & 7 & 0 & 1 \\ d & 6 & \infty & \infty & 0 \end{array}$$ → weight matrix.

$D[0] = W$ $D[1] = ?$ a $D[2] = ?$ a, b $D[3] = ?$ a, b, c

$D[4] =$ a, b, c, d.

↓ intermediate nodes.

$$D[0] = \begin{bmatrix} 0 & \infty & 3 & \infty \\ 2 & 0 & \infty & \infty \\ \infty & 7 & 0 & 1 \\ 6 & \infty & \infty & 0 \end{bmatrix}$$

$$D[1] = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & a & \infty & 3 & \infty \\ b & 2 & 0 & \boxed{5} & \infty \\ c & \infty & 7 & 0 & 1 \\ d & 6 & \infty & \boxed{9} & 0 \end{array}$$ with intermediate 'a'

$$D[2] = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & \infty & 3 & \infty \\ b & 2 & 0 & 5 & \infty \\ c & \boxed{9} & 7 & 0 & 1 \\ d & 6 & \infty & 9 & 0 \end{array}$$ with intermediate 'a' and 'b'.

$$D[3] = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & \boxed{10} & 3 & \boxed{4} \\ b & 2 & 0 & 5 & \boxed{6} \\ c & 9 & 7 & 0 & 1 \\ d & 6 & \boxed{16} & 9 & 0 \end{array}$$ with intermediate 'a', 'b' and 'c'.

$$D[4] = \begin{array}{c|cccc} & a & b & c & d \\ a & 0 & 10 & 3 & 4 \\ b & 2 & 0 & 5 & 6 \\ c & 7 & 7 & 0 & 1 \\ d & 6 & 16 & 9 & 0 \end{array}$$

$$D^{k}(i,j) = \text{Min}\left[D^{k-1}(i,j),\ D^{k-1}(i,k) + D^{k-1}(k,j) \right]$$

Algo:

```
D ← W
for k ← 1 to n do
{
for i ← 1 to n do
{
for j ← 1 to n do
D[i,j] = min { D[i,j] , D[i,k] + D[k,j] }
}
}
}
return D.
```

Analysis:

$$\sum_{k=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} 1$$

$$= \sum_{k=1}^{n} n^2 - 1$$

$c(n) = n^3$

$\theta(n^3)$.