Dijkstra's algorithm

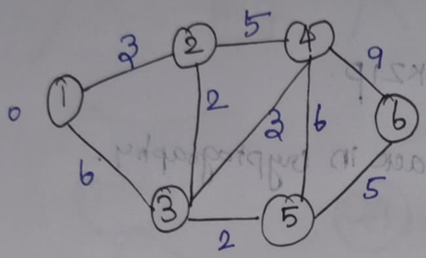
This algorithm is used to find the shortest path between two mentioned vertices of the given directed or undirected graph by using greedy technique.

Ex:- Shortest distance finding in google map.

The weight of the graph should be non-negative.

This is BFS - Breadth First Search.

→ Also called single source shortest path.



visited	unvisited	weights
{1}	{2,3,4,5,6}	{0,0,∞,∞,∞,∞}
{1,2}	{3,4,5,6}	{0,3,6,∞,∞,∞}
{1,2,3}	{4,5,6}	{0,3,5,8,∞,∞}
{1,2,3,5}	{4,6}	{0,3,5,8,7,∞}
{1,2,3,4,5}	{6}	{0,3,5,8,7,12}
{1,2,3,4,5,6}	-	{0,3,5,8,7,12}

Algorithm:-

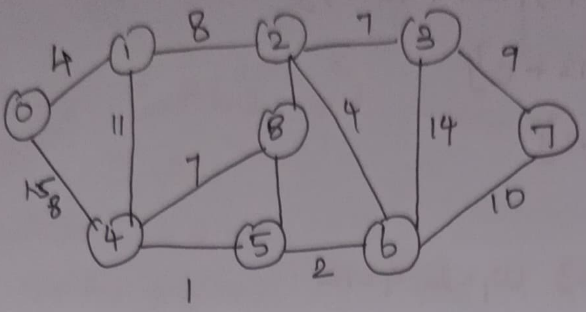
There are 'n' vertices in weighted directed graph, finding shortest path from vertex 'i' to 'j' n2 vertex pair (i,j).

Algorithm works 'n' times for each pair.

hence time complexity is $O(n^3)$ times.

or $O(VE)$

Ex:- 2 Find



Knapsack problem using Dynamic programming

Formula:-

$$F(i,j) = \begin{cases} \max\ F(i-1,j),\ v_i + F(i-1, j-w_i) & \text{if } j-w_i \geq 0 \\ F(i-1,j) & \text{if } j-w_i < 0 \end{cases}$$

Ex:-

Item	wt	Value
1	2	12
2	1	10
3	3	20
4	2	15

W = 5 capacity.

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	12	12	12	12
2	0	10	12	22	22	22
3	0	10	12	22	30	32
4	0	10	15	25	30	37

$F(1,1) = i=1 \quad j=1 \quad w_1 = 2 \quad v_i = (v_1) = 12$

$j - w_i = 1 - 2 = -1$

$F(1,1) = \max\ F(i-1,j) = F(1-1,1) = F(0,1) = 0$

$F(1,2) = i=1 \quad j=2 \quad w_i=2 \quad V_1=12$

$j-w_i \quad 2-2=0$

$F(1,2) = \max(F(i-1,j), V_i + F(i-1, j-w_i))$

$= \max\{F(0,2), 12+F(0,0)\}$

$= \max\{0, 12+0\}$

$= \max\{0,12\}$

$F(1,2) = 12$

$F(1,3) = i=1 \quad j=3 \quad w_1=2 \quad V_1=12$

$j-w_i = 3-2=1$

$F(1,3) = \max\{F(0,3), 12+F(0,1)\}$

$= \max\{0, 12+0\}$

$F(1,3) = 12$

Similarly $F(1,4)$ and $F(1,5) = 12$

$F(2,1) = 10$	$F(3,1) = 10$	$F(4,1) = 10$
$F(2,2) = 12$	$F(3,2) = 12$	$F(4,2) = 15$
$F(2,3) = 22$	$F(3,3) = 22$	$F(4,3) = 25$
$F(2,4) = 22$	$F(3,4) = 30$	$F(4,4) = 30$
$F(2,5) = 22$	$F(3,5) = 32$	$F(4,5) = 37.$

$F(2,5) = \quad i=2 \quad j=5 \quad w_2=1 \quad V_2=10$

$j-w_i \Rightarrow 5-1 \Rightarrow 4$

$F(2,5) = \max\{F(i-1,j), V_i + F(i-1, j-w_i)\}$

$= \max\{F(1,5), 10 + F(1,4)\}$

$= \max\{12, 10+12\}$

$= \max\{12, 22\}$

$F(2,5) = \underline{\underline{22}}.$

Selecting item

Take last value which is 4, 5

F(i, k) ≠ F(i-1, k) i=4, k=5 | F(i=k) = F(i-1, k)

select i | don't select i

i => i-1 k -> k-wi | i => i-1

1 2 3 | k => k

i k
F(4, 5) = F(3, 5)
i-1 k

37 ≠ 32

So, select i = 4 ✓

i = i-1 => 3

k = 5 - 2 = 3

wt of 4

1 -> 2
2 -> 1
3 -> 3
4 -> 2

Now i = 3 k = 3

F(3, 3) = F(2, 3)

22 = 22.

i = i-1 => 2 | i = 1
k = 3 ✓ | k = 3

F(2, 3) = F(1, 3) =>

=> F(2, 3) ≠ F(1, 3)

select 2 ✓

i = 1 ✓ | i = 0
k = 3-1 | k = 3-1
k = 2 |
F(1, 2) | F(0, 2)

=> F(1, 2) ≠ F(0, 2)

select 1 ✓

i => 0 | i = 0
k = 2-2 | k = 2-2 = 0
F(0, 0) | F(0, 0)

So, we can stop.

Selecting items :- c=5
{1, 2, 4}

vector =
{1, 1, 0, 1}

There is item for 0, 1, and 4.
No item for 3. so it is 0.