Assignment problems

We need to do exhaustive search. Task 'n' people to execute 'n' jobs. One person per job.
The problem is to find assignment with minimum cost of total.

Ex:

	Job1	Job2	Job3	Job4
Person1	9	2	7	8
Person2	6	4	3	7
Person3	5	8	1	8
person4	7	6	9	4

$\Rightarrow$ nb = 4! $\Rightarrow$ $4\times3\times2\times1 = 24$ possible ways.

↓ minimum optimal solution

which value is minimum, that is optimal solution.

24 $\Rightarrow$ 6x4. P-1 $\to$ b, P2 $\to$ b, $P_3 \to b$, P4 $\to$ b total 24.

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment	Cost
c(1,2,3,4)	9+4+1+4 = 18
c(1,2,4,3)	9+4+8+9 = 30
c(1,3,2,4)	9+3+8+4 = 24
c(1,3,4,2)	9+3+8+6 = 26
c(1,4,2,3)	9+7+8+9 = 33
c(1,4,3,2)	9+7+1+6 = 23
c(2,1,3,4)	3+6+1+4 = 13 — minimum value
c(2,1,4,3)	3+6+8+9 = 15
c(2,3,1,4)	2+3+5+4 = 14
c(2,3,4,1)	2+3+8+9 = 20
c(2,4,1,3)	2+9+5+9 = 33

b

c(3,1,2,4)	7+6+8+9=25
c(3,1,4,2)	7+6+8+4=27
c(3,2,1,4)	7+4+5+4=20
c(3,2,4,1)	7+4+8+7=26
c(3,4,1,2)	7+7+5+6=25
c(3,4,2,1)	7+7+8+7=29

b

c(4,1,2,3)	8+6+8+9=31
c(4,1,3,2)	8+6+1+6=21
c(4,2,1,3)	8+4+5+9=26
c(4,2,3,1)	8+4+1+7=20
c(4,3,1,2)	8+3+5+6=22
c(4,3,2,1)	8+3+8+7=26

b

Hence $\langle 2,1,3,4 \rangle = 18$ 13 which is optimal solution.

->Here we are generating all the permutations of the integers 1,2... n . n! times.

Hence time complexity is $\Omega(n!)$ or $\underline{\underline{O(n!)}}$.