Multiplication of larger integers

Multiplication :-

Multiply the multiplicand by each of multiplier and then add up all the property of shifted results. This is called " Grade school multiplication".

Ex!

$$\begin{array}{r} 42 \times \\ 34 \\ \hline 168 \\ 1260 \\ \hline 1428 \\ \hline \end{array}$$

Padding '0' at unit position.

Let $c = a * b \rightarrow ①$

$c = c_2 10^2 + c_1 10^1 + c_0 \rightarrow ②$

sub ① in ②

$a * b = c_2 10^2 + c_1 10^1 + c_0 \rightarrow ③$

where $c_2 = a_1 * b_1$, $c_1 = (a_1 + a_0) * (b_1 + b_0) - (c_2 + c_0)$

$c_0 = a_0 * b_0$.

Ex: 42×34

$a = a_1 a_0$ $a_1 = 4$ $a_0 = 2$

$b = b_1 b_0$ $b_1 = 3$ $b_0 = 4$.

To get values of c_0 c_1 and c_2, we need to calculate $c_0 = a_0 * b_0$

$c_0 = 2 \times 4$

$c_0 = 8$

$c_2 = a_1 * b_1$

$\Rightarrow 4 \times 3$

$c_2 = 12$

$\therefore c_1 = (4+2) * (3+4) - (12+8)$

$= (6) \times (7) - (20)$

$= 42 - 20$

$c_1 = 22$.

$\therefore$ Sub c_0, c_1, c_2 in ③

$a * b = 12 \times 10^2 + 22 \times 10^1 + 8$

$= 1200 + 220 + 8$

$\boxed{42 * 34 = 1428}$

Mathematical Analysis:

$c_0 = a_0 * b_0 \rightarrow$ mul ①

$c_1 = (a_1+a_0) * (b_1+b_0) - (c_2+c_0)$ mul ②

$c_2 = a_1 * b_1 \rightarrow$ mul ③

If there are 'n' digits to be multiplied. Then the multiplication of 'n' digit requires 3 multiplication.

$c(n) = 3c(n/2) \quad n>1$

$c(n) = 1 \quad n=1$

Assume $n=2^k$

$c(2^k) = 3c(2^k/n)$

$K = K-1 \quad \Rightarrow 3c(2^{k-1}$

$c(2^{k-1}) = 3c(2^{k-1}/2^k)$

$= 3c(2^{k-2})$

$c(2^{k-2}) = 3c(2^{k-2}/2]$

$= 3c[2^{k-3}]$

$c(2^k) = 3^k$

Taking log on both sides

$K = \log_2 n$

Strassen's matrix multiplication:-

If we want to multiply two matrices. A and B of size 'n'.

$C = A \times B$ then

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

$C_{11} = (A_{11} \times B_{11}) + (A_{11} \times B_{21})$

$C_{12} = (A_{11} \times B_{12}) + (A_{11} \times B_{22})$

$C_{21} = (A_{21} \times B_{11}) + (A_{22} \times B_{21})$

$C_{22} = (A_{21} \times B_{12}) + (A_{22} \times B_{22})$

The multiplication requires 8 multiplication.

This operations are costly.

So, we move to Strassen's multiplication.

↓

requires 7 multiplication.

$$\begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \times \begin{pmatrix} B_{21} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$$

$S_1 = (A_{11} + A_{22}) \times (B_{11} + B_{22})$

$S_2 = (A_{21} + A_{22}) \times B_{11}$

$S_3 = A_{11} \times (B_{12} - B_{22})$

$S_4 = A_{22} \times (B_{21} - B_{11})$

$S_5 = (A_{11} + A_{12}) \times B_{22}$

$S_6 = (A_{21} - A_{11}) \times (B_{11} + B_{12})$

$S_7 = (A_{12} - A_{22}) \times (B_{22} + B_{22})$

5+2+18+48

$C_{11} = S_1 + S_4 - S_5 + S_7$

$C_{12} = S_3 + S_5$

$C_{21} = S_2 + S_4$

$C_{22} = S_1 + S_3 - S_2 + S_6$.

Ex: $A = \begin{array}{c} \begin{array}{cc} A_{11} & 12 \end{array} \\ \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix} \\ \begin{array}{cc} 21 & 22 \end{array} \end{array}$ $B = \begin{array}{c} \begin{array}{cc} B_{11} & 12 \end{array} \\ \begin{bmatrix} 6 & 7 \\ 3 & 8 \end{bmatrix} \\ \begin{array}{cc} 21 & 22 \end{array} \end{array}$

$S_1 = (A_{11} + A_{22}) \times (B_{11} + B_{22})$

$= (1 + 5) \times (6 + 8)$

$= 6 \times 14$

$S_{11} = 84$

$S_2 = (A_{21} + A_{22}) \times B_{11}$

$= (7 + 5) \times 6$

$= 12 \times 6$

$S_2 = 72$

$14 \times 6 = 84$

$S_3 = A_{11} \times (B_{12} - B_{22})$

$= 1 \times (7 - 8)$

$= 1 \times -1$

$S_3 = -1$

$S_4 = A_{22} \times (B_{21} - B_{11})$

$= 8 \times (3 - 6)$

$= 8(-3) = -24$

$S_4 = -24$

$S_5 = (A_{11} + A_{12}) \times B_{22}$

$= (1+3) \times (8)$

$= 4 \times 8$

$S_5 = 32$

$S_6 = (A_{21} - A_{11}) \times (B_{11} + B_{12})$

$= (7-1) \times (6+7)$

$= 6 * 13$

$= 78$

$S_7 = (A_{12} - A_{22}) \times (B_{21} + B_{22})$

$= (3-5) \times (3+8)$

$= -2 \times 11$

$= -22$

$S_1 = 84$
$S_2 = 72$
$S_3 = -1$
$S_4 = -24$
$S_5 = 32$
$S_6 = 78$
$S_7 = -22$

$C_{11} = S_1 + S_4 - S_5 + S_7$

$C_{12} = S_3 + S_5$

$C_{21} = S_2 + S_4$

$C_{22} = S_1 + S_3 - S_2 + S_6$

$C_{11} = 84 - 24 - 32 + (-22)$

$= 6$

$C_{12} = -1 + 32$

$= 31$

$C_{21} = 72 - 24$

$= 48$

$C_{22} = 84 - 1 - (-72) + 78$

$= 233$

Algorithm::

$$T(n) = \begin{cases} C \\ 7 \times T(n/2) + d \times n^2 & \text{if } n=1 \end{cases}$$

$$T(n) = O(n^{\log 7})$$

Closest pair using Divide and Conquer::

we are given array of 'n' points in the plane and we need to find the closest of each plane, we need use algorithm.

This problem arises in the traffic control of planes

By using Brute force, complexity is $O(n \log n)$