Here there are 3 pairs of tower.

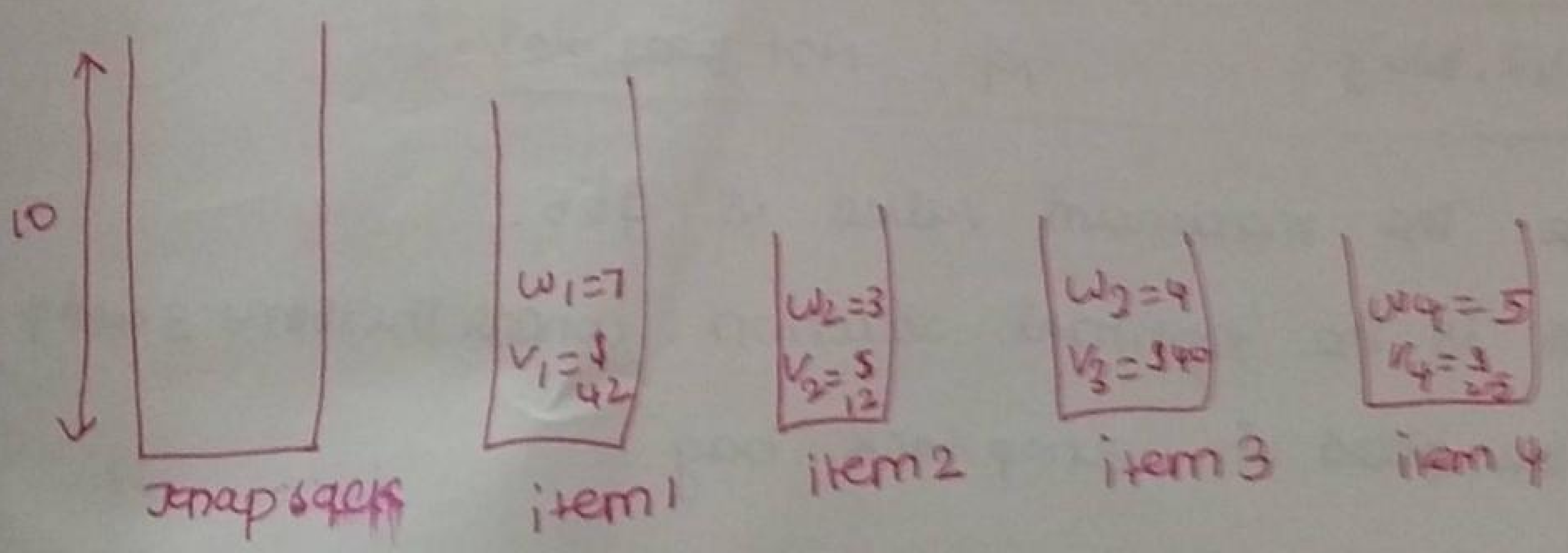
Knapsack problem

We are given a knapsack or a bag that can hold weight upto a certain value. Each items will have different weights and values associated with them. Now we are in need to fill the knapsack such that sum of total weight of filled items does not exceed the maximum capacity.

$\boxed{\text{Knapsack}}$ maximum weight limit = 30

item 1	item 2	item 3
weight = 5 units	weight = 20 units	weight = 10 units
value = 50	value = 140	value = 60

Example:



Subset	Total weight	Total value
0	0	\$0
1	7	\$42
2	3	\$12
3	4	\$40
4	5	\$25

Solution

Subset	Total weight	Total value
ϕ	0	$0
{1}	7	$42
{2}	3	$12
{3}	4	$40
{4}	5	$25
{1,2}	10	$36
{1,3}	11	not feasible
{1,4}	12	not feasible
{2,3}	7	$52
{2,4}	8	$37
{3,4}	9	$65
{1,2,3}	14	Not feasible
{1,2,4}	15	not feasible
{1,3,4}	16	not feasible
{2,3,4}	12	not feasible
{1,2,3,4}	19	Not feasible.

here the maximum value is $65.

This is the optimal solution. where the items 3 and 4 can be filled in knap sack bag.

Time complexity:

This problem uses the exhaustive search. This leads to the generation of subsets from the set of 'n' items.

The no. of subsets of n items is 2^n.

hence time complexity can be $\Omega(2^n)$

Assignment problems

we need to do exhaustive search for 'n' people to execute 'n' jobs. One person per job

The problem is to find assignment with minimum cost of total.

Ex:-

	Job1	Job2	Job3	Job4
Person1	9	2	7	8
Person2	6	4	3	7
Person3	5	8	1	8
Person4	7	6	9	4

$\Rightarrow n! = 4! \Rightarrow 4\times3\times2\times1 = 24$ possible ways.

↓

minimum optimal solution.

which value is minimum, that is optimal solution.

$24 \Rightarrow 6\times4$. $P_1 \to 6$, $P_2 \to 6$, $P_3 \to 6$, $P_4 \to 6$ total 24.

$$C = \begin{bmatrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{bmatrix}$$

Assignment	cost
c(1,2,3,4)	9+4+1+4 = 18
c(1,2,4,3)	9+4+8+9 = 30
c(1,3,2,4)	9+3+8+4 = 24
c(1,3,4,2)	9+3+8+6 = 26
c(1,4,2,3)	9+7+8+9 = 33
c(1,4,3,2)	9+7+1+6 = 23

6

c(2,1,3,4)	3+6+1+4 = 13	minimum value
c(2,1,4,3)	3+6+8+9 = 15	
c(2,3,1,4)	2+3+5+4 = 14	
c(2,3,4,1)	2+3+8+9 = 20	
c(2,4,1,3)	2+9+5+9 = 33	
c(2,4,3,1)	2+7+1+7 = 17.	

6

c(3,1,2,4) 7+6+8+9=25

c(3,1,4,2) 7+6+8+4=27

c(3,2,1,4) 7+4+5+4=20

c(3,2,4,1) 7+4+8+7= 26

c(3,4,1,2) 7+7+5+6=25

c(3,4,2,1) 7+7+8+7=29

6

c(4,1,2,3) 8+6+8+9=31

c(4,1,3,2) 8+6+1+6=21

c(4,2,1,3) 8+4+5+9=26

c(4,2,3,1) 8+4+1+7=20

c(4,3,1,2) 8+3+5+6=22

c(4,3,2,1) 8+3+8+7=26

6

Hence <2,1,3,4> = ~~18~~ 13 which is optimal solution

-> Here we are generating all the permutations of the integers 1,2.... n. n! times.

Hence time complexity is $\Omega(n!)$ or $O(n!)$.