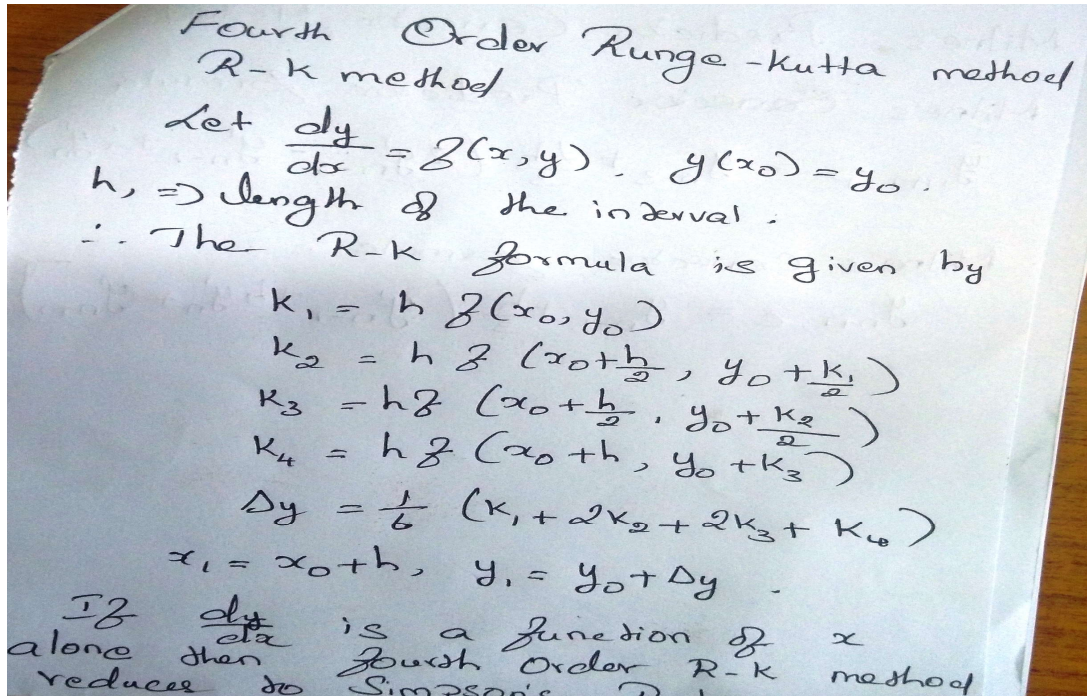




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Topic: 5.7 – Fourth order Runge-Kutta method for solving first order equations





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3. Find $y(0.8)$ given that $y' = y - x^2$, $y(0.6) = 1.7379$, by using Runge-Kutta method of fourth order. Take $h = 0.1$. [AU M/J '2012]

Soln:

Given $y' = f(x, y) = y - x^2$, $x_0 = 0.6$, $y_0 = 1.7379$.

$x_1 = 0.7$, $x_2 = 0.8$, $h = 0.1$.

To find $y(0.7)$

$K_1 = h f(x_0, y_0) = (0.1) [y_0 - x_0^2] = (0.1) [1.7379 - (0.6)^2] = 0.13779$.

$K_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}) = 0.1 f(0.6 + \frac{0.1}{2}, 1.7379 + \frac{0.13779}{2})$
 $= 0.1 f(0.65, 1.8086795)$
 $= (0.1) [1.8086795 - (0.65)^2]$
 $= 0.13843$.

$K_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}) = 0.1 f(0.65, 1.7379 + \frac{0.13843}{2})$
 $= 0.1 f(0.65, 1.807115)$
 $= (0.1) [1.807115 - (0.65)^2] = 0.13846$.

$K_4 = h f(x_0 + h, y_0 + K_3) = 0.1 f(0.6 + 0.1, 1.7379 + 0.13846)$
 $= 0.1 f(0.7, 1.87636)$

$K_3 = \dots$
 $= 0.1 f(0.65, 1.807115)$
 $= (0.1) [1.807115 - (0.65)^2] = 0.13846$

$K_4 = h f(x_0 + h, y_0 + K_3) = 0.1 f(0.6 + 0.1, 1.7379 + 0.13846)$
 $= 0.1 f(0.7, 1.87636)$
 $= (0.1) [1.87636 - (0.7)^2]$
 $= 0.13864$.

$\Delta y = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$
 $= \frac{1}{6} [0.13779 + 2(0.13843) + 2(0.13846) + 0.13864]$
 $= 0.13837$.

$\therefore y_1 = y(0.7) = y_0 + \Delta y = 1.7379 + 0.13837 = 1.87627$.

$y_1 = 1.876$ (app)

To find $y(0.8)$