



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Topic: 5.3 – Taylor’s Series Method

$y(0.8) = 1.6692$

b. Using Taylor's method, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ if

$\frac{dy}{dx} = x - y^2, y(0) = 1.$ [Nov/Dec '12]

Soln:

Given $x_0 = 0, x_1 = 0.1, x_2 = 0.2, x_3 = 0.3.$
 $y_0 = 1, h = 0.1.$

To find $y(0.1)$

Taylor's series expansion is
 $y_1 = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \frac{h^4}{4!} y_0^{(4)} + \dots$

$y' = x - y^2$

$y'' = 1 - 2yy'$

$y''' = -2[y_0 y_0'' + y_0'^2]$

$y_0' = x_0 - y_0^2 = -1$

$y_0'' = 1 - 2y_0 y_0' = 3$

$y_0''' = -2[y_0 y_0'' + y_0'^2] = -8$

$y_1 = y(0.1) = 1 + \frac{(0.1)^1}{1!} (-1) + \frac{(0.1)^2}{2!} (3) + \frac{(0.1)^3}{3!} (-8) + \dots$



SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

$$= 1 - 0.1 + 0.0051 + 0.0113$$

$$= 0.9137$$

To find $y(0.2)$.

$$x_1 = 0.1, \quad y_1 = 0.9137$$

Taylor's series expansion is

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$$

$$y_1' = x_1 - y_1^2 = 0.1 - (0.9137)^2 = -0.7348$$

$$y_1'' = 1 - 2y_1 y_1' = 1 - 2(0.9137)(-0.7348) = 2.3428$$

$$y_1''' = -2[y_1 y_1'' + y_1'^2]$$

$$= -2[(0.9137)(2.3428) + (-0.7348)^2] = -5.3611$$

$$\therefore y_2 = y(0.2) = 0.9137 + (0.1)(-0.7348) + \frac{(0.1)^2}{2!}(2.3428) + \frac{(0.1)^3}{3!}(-5.3611)$$

$$= 0.9137 - 0.07348 + 0.0117 - 0.0009$$

$$y(0.2) = 0.8510$$

(99)



SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

$$= 1 - 0.1 + 0.0051 + 0.0113$$

$$= 0.9137$$

To find $y(0.2)$.

$$x_1 = 0.1, y_1 = 0.9137.$$

Taylor's series expansion is

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$$

$$y_1' = x_1 - y_1^2 = 0.1 - (0.9137)^2 = -0.7348$$

$$y_1'' = 1 - 2y_1 y_1' = 1 - 2(0.9137)(-0.7348) = 2.3428.$$

$$y_1''' = -2 [y_1 y_1'' + y_1'^2]$$

$$= -2 [(0.9137)(2.3428) + (-0.7348)^2] = -5.3611$$

$$\therefore y_2 = y(0.2) = 0.9137 + (0.1)(-0.7348) + \frac{(0.1)^2}{2!}(2.3428) + \frac{(0.1)^3}{3!}(-5.3611)$$

$$= 0.9137 - 0.07348 + 0.0117 - 0.0009$$

$$y(0.2) = 0.8510$$

(99)