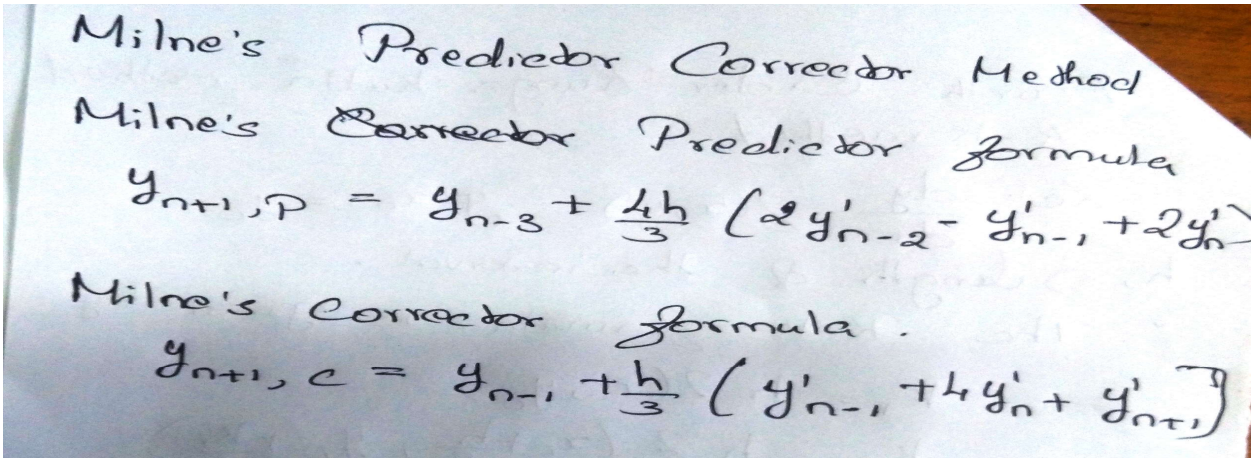
Topic: 5.11 – Milnes Predictor Corrector Formula



PREDICTOR - CORRECTOR METHODS

PREDICTOR FORMULA :

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

CORRECTOR FORMULA

$$y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right)$$

2. Using Milne's Predictor – corrector method, find $y(4.4)$ given $5xy' + y^2 - 2 = 0$; $y(4) = 1$, $y(4.1) = 1.0069$, $y(4.2) = 1.0097$, $y(4.3) = 1.01$ [AU M/J '12, M/J

Soln:

Milne's predictor - corrector formula is

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3}(2y'_{n-2} - y'_{n-1} + 2y'_n)$$

$$y_{4,p} = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3) \quad \text{---(1)}$$

Given $5xy' + y^2 - 2 = 0$

$5xy' = 2 - y^2$

$y' = \frac{2 - y^2}{5x}$

$x_0 = 4$, $x_1 = 4.1$, $x_2 = 4.2$, $x_3 = 4.3$

$y_0 = 1$, $y_1 = 1.0049$, $y_2 = 1.0097$, $y_3 = 1.0143$

$$y'_1 = \frac{2 - y_1^2}{5x_1} = \frac{2 - (1.0049)^2}{5(4.1)} = 0.0493.$$

$$y'_2 = \frac{2 - y_2^2}{5x_2} = \frac{2 - (1.0097)^2}{5(4.2)} = 0.0467.$$

$$y'_3 = \frac{2 - y_3^2}{5x_3} = \frac{2 - (1.0143)^2}{5(4.3)} = 0.0452.$$

Sub all these values in (1),

$$y_{4,p} = 1 + \frac{4(0.1)}{3}[2(0.0493) - 0.0467 + 2(0.0452)]$$

$$= 1.01897.$$

$$y'_4 = \frac{2 - y_4^2}{5(x_4)} = \frac{2 - (1.01897)^2}{5(4.4)} = 0.0437.$$

Using Milne's corrector formula,

$$y_{4,c} = y_2 + \frac{h}{3}[y'_2 + 4y'_3 + y'_4]$$

$$= 1.0097 + \frac{0.1}{3}[0.0467 + 4(0.0452) + 0.0437]$$

$$y_{4,c} = 1.01874.$$

$\therefore \boxed{y(4.4) = 1.01874}$