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Topic: 5.9 – Fourth order Runge-Kutta method for solving first order equations

Again apply Runge - Kutta Method,

$$x_1 = 0.7, \quad y_1 = 1.876$$

$$K_1 = h f(x_1, y_1) = (0.1) f(0.7, 1.876) \\ = (0.1) [1.876 - (0.7)^2] = 0.1386$$

$$K_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right) = (0.1) f\left[0.7 + \frac{0.1}{2}, 1.876 + \frac{0.1386}{2}\right] \\ = (0.1) f(0.75, 1.9453) \\ = (0.1) [1.9453 - (0.75)^2] \\ = 0.13828$$

$$K_3 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_3}{2}\right) \\ = (0.1) f\left[0.75, 1.876 + \frac{0.13828}{2}\right] \\ = (0.1) f(0.75, 1.94514) \\ = (0.1) [1.94514 - (0.75)^2] = 0.138264$$

$$K_4 = h f(x_1 + h, y_1 + K_3) \\ = (0.1) f(0.7 + 0.1, 1.876 + 0.138264) \\ = (0.1) f(0.8, 2.014264) \\ = (0.1) [2.014264 - (0.8)^2] \\ = 0.1374264$$

$$\Delta y = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4] \\ = \frac{1}{6} [0.1386 + 2(0.13828) + 2(0.138264) + 0.1374264] \\ = 0.138186$$

$$y_2 = y(0.8) = y_1 + \Delta y = 1.876 + 0.138186 = 2.014186 \\ \boxed{y(0.8) = 2.0142} \text{ (correct to 4 decimal places)}$$

The Solution is ..

5. Given $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$, $y(0) = 1$, find $y(0.2)$, $y(0.4)$, and $y(0.6)$ by R. Kutta method of fourth order. Find $y(0.8)$ by Milne's predictor - corrector method. [November 2012]

Soln:

$$\text{Given } x_0 = 0, \quad x_1 = 0.2, \quad x_2 = 0.4, \quad x_3 = 0.6, \quad x_4 = 0.8 \\ y_0 = 1$$

To find $y(0.2)$:



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$$x_0 = 0, y_0 = 1, h = 0.2$$

$$\frac{dy}{dx} = f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

$$K_1 = h f(x_0, y_0) = 0.2 f(0, 1) = 0.2 \left[\frac{1^2 - 0^2}{1^2 + 0^2} \right] = 0.2$$

$$K_2 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right) = 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right) \\ = 0.2 f(0.1, 1.1) = (0.2) \left[\frac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2} \right] = 0.1967$$

$$K_3 = h f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= (0.2) f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1967}{2}\right)$$

$$= (0.2) f(0.1, 1.0984)$$

$$= (0.2) \left[\frac{(1.0984)^2 - (0.1)^2}{(1.0984)^2 + (0.1)^2} \right] = 0.1967$$

$$K_4 = h f(x_0 + h, y_0 + K_3) = (0.2) f(0.2, 1.1967)$$

$$= (0.2) \left[\frac{(1.1967)^2 - (0.2)^2}{(1.1967)^2 + (0.2)^2} \right] = 0.1891$$

$$\Delta y = \frac{1}{6} (K_1 + 2K_2 + 2K_3 + K_4)$$

$$= \frac{1}{6} (0.2 + 2(0.1967) + 2(0.1967) + 0.1891)$$

$$= 0.1960$$

$$y(0.2) = y_1 = y_0 + \Delta y = 1 + 0.1960 = 1.1960$$

$$\boxed{y(0.2) = 1.1960}$$

To find $y(0.4)$

$$x_1 = 0.2, y_1 = 1.1960, h = 0.2$$

$$K_1 = h f(x_1, y_1) = 0.2 f(0.2, 1.1960) = (0.2) \left[\frac{(1.1960)^2 - (0.2)^2}{(1.1960)^2 + (0.2)^2} \right]$$

$$= 0.1891$$

$$K_2 = h f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$$



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$$\begin{aligned} &= 0.2 \frac{1}{6} \left(0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1891}{2} \right) \\ &= 0.2 \frac{1}{6} (0.3, 1.2906) \\ &= 0.2 \left[\frac{(1.2906)^2 - (0.3)^2}{(1.2906)^2 + (0.3)^2} \right] \\ &= 0.1795 \\ K_3 &= h \frac{1}{6} \left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2} \right) \\ &= 0.2 \frac{1}{6} \left(0.2 + \frac{0.2}{2}, 1.196 + \frac{0.1795}{2} \right) \\ &= 0.2 \frac{1}{6} (0.3, 1.1858) \\ &= 0.2 \left[\frac{(1.1858)^2 - (0.3)^2}{(1.1858)^2 + (0.3)^2} \right] = 0.1793 \end{aligned}$$

$$\begin{aligned} K_4 &= h \frac{1}{6} (x_1 + th, y_1 + K_3) = (0.2) \frac{1}{6} (0.4, 1.3753) \\ &= (0.2) \left[\frac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2} \right] = 0.1688 \\ \Delta y &= \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4] \\ &= \frac{1}{6} (0.1891 + 2(0.1795) + 2(0.1793) + 0.1688) \\ &= 0.1793 \\ y(0.4) &= y_2 = y_1 + \Delta y = 1.196 + 0.1793 \\ &\boxed{y(0.4) = 1.3753} \end{aligned}$$

To find $y(0.6)$.

$$x_2 = 0.4, y_2 = 1.3753, h = 0.2$$

$$\begin{aligned} K_1 &= (0.2) \frac{1}{6} (x_2, y_2) = (0.2) \frac{1}{6} (0.4, 1.3753) \\ &= (0.2) \left[\frac{(1.3753)^2 - (0.4)^2}{(1.3753)^2 + (0.4)^2} \right] = 0.1688 \end{aligned}$$

$$\begin{aligned} K_2 &= h \frac{1}{6} \left(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2} \right) \\ &= (0.2) \frac{1}{6} \left(0.4 + \frac{0.2}{2}, 1.3753 + \frac{0.1688}{2} \right) \end{aligned}$$



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$$= 0.2 \frac{1}{b} (0.5, 1.4597)$$
$$= 0.2 \left[\frac{(1.4597)^2 - (0.5)^2}{(1.4597)^2 + (0.5)^2} \right] = 0.1580$$

$$K_3 = h \frac{1}{b} \left(x_2 + \frac{h}{2}, y_2 + \frac{K_2}{2} \right)$$
$$= 0.2 \frac{1}{b} \left(0.4 + \frac{0.2}{2}, 1.3753 + \frac{0.1580}{2} \right)$$
$$= 0.2 \frac{1}{b} (0.5, 1.4543)$$
$$= 0.2 \left[\frac{(1.4543)^2 - (0.5)^2}{(1.4543)^2 + (0.5)^2} \right] = 0.1577$$

$$K_4 = h \frac{1}{b} (x_3 + h, y_3 + K_3)$$
$$= (0.2) \frac{1}{b} (0.4 + 0.2, 1.3753 + 0.1577)$$
$$= (0.2) \frac{1}{b} (0.6, 1.5330)$$
$$= 0.2 \left[\frac{(1.5330)^2 - (0.6)^2}{(1.5330)^2 + (0.6)^2} \right] = 0.1469$$

$$\Delta y = \frac{1}{b} [K_1 + 2K_2 + 2K_3 + K_4]$$
$$= \frac{1}{b} [0.1688 + 2(0.1580) + 2(0.1577) + 0.1469]$$
$$= 0.1578$$

$$y(0.6) = y_3 = y_2 + \Delta y = 1.3753 + 0.1578$$

$$\boxed{y(0.6) = 1.5331}$$