Topic: 5.5 – **Euler's & Modified Euler's method**

Euler's Method:

The general formula is given by

$$y_{n+1} = y_n + h \, f(x_n, y_n)$$

where $n = 0, 1, 2 \cdots$

Modified Euler's Method

The Modified Euler's algorithm is given by

$$y_{n+1} = y_n + h \, f\left[x_n + \frac{h}{2}, \; y_n + \frac{h}{2} f(x_n, y_n)\right]$$

where $n = 0, 1, 2 \cdots$
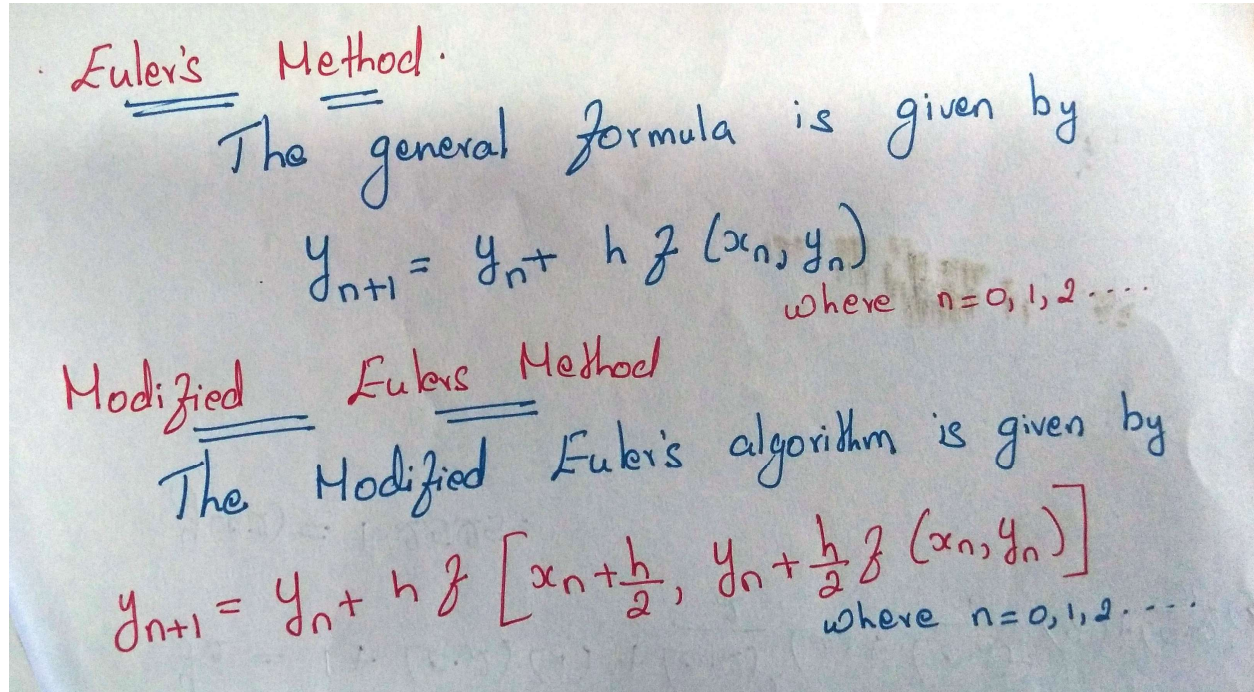
7. Consider the initial value problem $\frac{dy}{dx} = y - x^2 + 1$, $y(0) = 0.5$. Find $y(0.2)$ by Euler's method and modified Euler's method [NOV/Dec'14]

Soln:

Given $x_0 = 0$, $x_1 = 0.2$

$y_0 = 0.5$ $h = 0.2$

$\frac{dy}{dx} = f(x, y) = y - x^2 + 1$.

Euler's formula is $y_{n+1} = y_n + h f(x_n, y_n)$; $n = 0, 1, 2, \ldots$

To find $y(0.2)$.

Here $x_0 = 0$, $y_0 = 0.5$, $h = 0.2$.

$y_1 = y_0 + h f(x_0, y_0)$

$= 0.5 + (0.2) f(0, 0.5)$

$= 0.5 + (0.2) [0.5 - 0^2 + 1]$

$y(0.2) = 0.8$

ie) $\boxed{y(0.2) = 0.8}$

Modified Euler's formula is

$y_{n+1} = y_n + h f [x_n + \frac{h}{2}, y_n + \frac{h}{2} f(x_n, y_n)]$; $n = 0, 1, 2 \ldots$

To find $y(0.2)$.

$x_0 = 0$, $y_0 = 0.5$, $h = 0.2$

$y_1 = y_0 + h f [x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)]$

$= 0.5 + (0.2) f [0 + \frac{0.2}{2}, 0.5 + \frac{0.2}{2} f(0, 0.5)]$

$= 0.5 + (0.2) f(0.1, 0.65)$.

$= 0.5 + (0.2) [0.65 - (0.1)^2 + 1]$

$= 0.828$

$\boxed{y(0.2) = 0.828}$