Topic: 5.3 – Taylor's Series Method

$y(0.8) = 1.6692$

5. Using Taylor's method, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ [Nov/Dec'12]

$\frac{dy}{dx} = x - y^2$, $y(0) = 1$.

Soln:

Given $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$.

$y_0 = 1$, $h = 0.1$.

To find $y(0.1)$

Taylor's series expansion is

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{iv} + \cdots\cdots$$

$y' = x - y^2$

$y'' = 1 - 2yy'$

$y''' = -2[yy'' + y'^2]$

$y_0' = x_0 - y_0^2 = -1$

$y_0'' = 1 - 2y_0y_0' = 3$

$y_0''' = -2[y_0y_0'' + y_0'^2] = -8$

$\therefore y_1 = y(0.1) = 1 + \frac{(0.1)(-1)}{1!} + \frac{(0.1)^2}{2!}(3) + \frac{(0.1)^3}{3!}(-8) + \cdots\cdots$

(99)

$= 1 - 0.1 + 0.0051 + 0.0113$

$= 0.9137$

To find $y(0.2)$.

$x_1 = 0.1, \quad y_1 = 0.9137$

Taylor's series expansion is

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$$

$y_1' = x_1 - y_1^2 = 0.1 - (0.9137)^2 = -0.7348$

$y_1'' = 1 - 2y_1 y_1' = 1 - 2(0.9137)(-0.7348) = 2.3428$

$y_1''' = -2[y_1 y_1'' + y_1'^2]$

$= -2[(0.9137)(2.3428) + (-0.7348)^2] = -5.3611$

$\therefore y_2 = y(0.2) = 0.9137 + (0.1)(-0.7348) + \frac{(0.1)^2}{2!}(2.3428) + \frac{(0.1)^3}{3!}(-5.3611)$

$= 0.9137 - 0.07348 + 0.0117 - 0.0009$

$\boxed{y(0.2) = 0.8510}$

$= 1 - 0.1 + 0.0051 + 0.0113$

$= 0.9137$

To find $y(0.2)$.

$x_1 = 0.1, \quad y_1 = 0.9137$

Taylor's series expansion is

$$y_2 = y_1 + \frac{h}{1!} y_1' + \frac{h^2}{2!} y_1'' + \frac{h^3}{3!} y_1'''$$

$y_1' = x_1 - y_1^2 = 0.1 - (0.9137)^2 = -0.7348$

$y_1'' = 1 - 2y_1 y_1' = 1 - 2(0.9137)(-0.7348) = 2.3428$

$y_1''' = -2[y_1 y_1'' + y_1'^2]$

$= -2[(0.9137)(2.3428) + (-0.7348)^2] = -5.3611$

$\therefore y_2 = y(0.2) = 0.9137 + (0.1)(-0.7348) + \frac{(0.1)^2}{2!}(2.3428) + \frac{(0.1)^3}{3!}(-5.3611)$

$= 0.9137 - 0.07348 + 0.0117 - 0.0009$

$\boxed{y(0.2) = 0.8510}$