Topic: 5.2– **Taylor's series method**

8. Given $\frac{dy}{dx} = x - y^2$, $y(0) = 1$, $y(0.1) = 0.9052$, $y(0.2) = 0.8213$, find $y(0.3)$ using Taylor's method. (NOV/DEC 2013). (10)

Soln: Here $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$

$y_0 = 1$, $y_1 = 0.9052$, $y_2 = 0.8213$, $h = 0.1$.

To find $y(0.3)$.

Here $x_2 = 0.2$, $y_2 = 0.8213$.

Taylor's Series is

$$y_3 = y(0.3) = y_2 + \frac{h}{1!}y_2' + \frac{h^2}{2!}y_2'' + \frac{h^3}{3!}y_3''' + \cdots$$

$y' = x - y^2$

$y'' = 1 - 2yy'$

$y''' = -2[yy'' + y'^2]$

$y_2' = x_2 - y_2^2 = -0.47$

$y_2'' = 1 - 2y_2y_2' = 1.77$

$y_2''' = -2[y_2y_2'' + y_2'^2] = -3.35$

$$y_3 = y(0.3) = 0.8213 + (0.1)(-0.47) + \frac{(0.1)^2}{2!}(1.77) + \frac{(0.1)^3}{3!}(-3.35)$$

$$= 0.8213 - 0.047 + 0.01 - 0.0006$$

$$\boxed{y(0.3) = 0.7837.}$$

$y(0.8) = 1.6692$

5. Using Taylor's method, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ if $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. [Nov/Dec'12]

Soln:

Given $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$.

$y_0 = 1$, $h = 0.1$.

To find $y(0.1)$

Taylor's series expansion is

$$y_1 = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + \frac{h^4}{4!}y_0^{iv} + \cdots\cdots$$

$y' = x - y^2$

$y'' = 1 - 2yy'$

$y''' = -2[yy'' + y'^2]$

$y_0' = x_0 - y_0^2 = -1$

$y_0'' = 1 - 2y_0y_0' = 3$

$y_0''' = -2[y_0y_0'' + y_0'^2] = -8$

$\therefore y_1 = y(0.1) = 1 + \frac{(0.1)(-1)}{1!} + \frac{(0.1)^2}{2!}(3) + \frac{(0.1)^3}{3!}(-8) + \cdots\cdots$