19MA407 - SNM

IAE - III

ANSWER KEY

PART A

1) i) $y'(x) = \frac{1}{h}\left\{ \Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^3 y_0 + \frac{4n^3 - 18n^2 + 22n - 6)}{24}\Delta^4 y_0 + \cdots \right\}$

ii) $y''(x) = \frac{1}{h^2}\left[\Delta^2 y_0 + (n-1)\Delta^3 y_0 + \frac{(6n^2-18n+11)}{12}\Delta^4 y_0 + \cdots\right]$.

2) $\int_{1/2}^{1} 1/x \, dx$. by Trapezoidal Rule.

$n = 4$, $h = 0.125$

x	0.5	0.625	0.75	0.875	1
$y = 1/x$	2	1.6	1.33	1.14	1

$$\int_{0.5}^{1} 1/x \, dx = \frac{h}{2}\left[[y_0 + y_4] + 2[y_1 + y_2 + y_3]\right]$$

$$= 0.697.$$

3) Taylor Series Alogorithm for 1st order DE

$y(x_0+h) = y_0 + \frac{h\,y'(0)}{1!} + \frac{h^2 y''(0)}{2!} + \frac{h^3 y'''(0)}{3!} + \cdots$

R.K. formula of 4th order.

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + h/2, y_0 + k_1/2)$

$k_3 = hf(x_0 + h/2, y_0 + k_2/2)$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$y_1 = y_0 + \Delta y$

5) Milne's Predictor formula:

$y_{n+1, P} = y_{n-3} + \frac{4h}{3}[2y'_{n-2} - y'_{n-1} + 2$

Milne's corrector formula:

$y_{n+1, C} = y_{n-1} + \frac{h}{3}[y'_{n-1} + 4y'_n + y'_{n+1}$

PART - B

1) To find $\log_{10} 656$, by Lagrange's method.

$x_0 = 654$ $y_0 = 2.8156$

$x_1 = 658$ $y_1 = 2.8182$

$x_2 = 659$ $y_2 = 2.8189$

$x_3 = 661$ $y_3 = 2.8202$

$$y = y_0 + \frac{h y_0'}{1!} + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \dots$$

$y' = x + y$	$y_0' = 0+1 = 1$	$y_1' = 1.215$
$y'' = y'' \cdot y'$	$y_0'' = 2$	$y_1'' = 2.355$
$y''' = y \cdot y'' + (y')^2$	$y_0''' = 3$	$y_1''' = 4.102$

$y_1(0.1) = 1.1115$ //

$y_2(0.2) = 8.0835$ //

$\frac{dy}{dt} \neq$

Qn: $dy/dx = y - x^2$, $y(0)=1$, $y(0.2) = 1.2186$, $y(0.4) = 1.46820$, $y(0.6) = 1.7379$

Find $y(0.8)$ by Milne's Predictor - Corrector Method.

Milne's Predictor :

$$y_{n+1}, P = y_{n-1} + 4\frac{h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$n = 3$

$$y_{n+1}, P = y_4 = 1.9630$$