SNS COLLEGE OF ENGINEERING

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AN AUTONOMOUS INSTITUTION

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Topic: 4.11 – Simpsons Rule

1). Evaluate $\int_0^1 e^{-x^2} dx$ by dividing the range of integration into 6 equal parts using Simpson's rule and Trapezoidal rule.

2). Evaluate $\int_1^2 \frac{\sin x}{x} dx$ by dividing the range into 6 equal parts using Simpson's rule and Trapezoidal rule.

Approximation of derivatives using interpolation polynomials

Forward Difference formula.

At $x = x_0$.

$$f'(x_0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 \cdots\right]$$

$$f''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \cdots\right]$$

$$f'''(x_0) = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$ and so on

At $x \neq x_0$.

$$f'(x_0) = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3-18u^2+22u-6}{24}\right)\Delta^4 y_0 + \cdots\right]$$

$$r = \frac{x_{[illegible]} - x_{[illegible]}}{h}$$

$$f''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \left(\frac{6u^2-18u+11}{12}\right)\Delta^4 y_0 + \cdots\right]$$

$$f'''(x_0) = \Delta^3 y_0 + \frac{2u-3}{2} \Delta^4 y_0 + \cdots$$

Backward Difference Formula

At $x = x_0$

$$f'(x_0) = \frac{1}{h}\left[\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^3 y_0 + \frac{1}{4}\nabla^4 y_0 - \cdots\right]$$

$$f''(x_0) = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0 + \cdots\right]$$

$$f'''(x_0) = \frac{1}{h^3}\left[\nabla^3 y_0 + \frac{3}{2}\nabla^4 y_0 + \cdots\right]$$

At $x \neq x_0$ $\qquad x+uh = x_0$

$$f'(x_0) = \nabla y_0 + \left(\frac{2u+1}{2}\right)\nabla^2 y_0 + \left(\frac{3u^2+6u+2}{6}\right)\nabla^3 y_0 + \frac{2u^3+9u^2+11u+3}{12}\nabla^4 y_0 + \cdots$$

$$u = \frac{x_0 - x}{h}$$