SNS COLLEGE OF ENGINEERING

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Topic: 4.10 – Trapezoidal Rule

Numerical Integration.

Trapezoidal Rule (1/3 Rule)

Formule:

$$\int_{x_0}^{x_0+nh} y(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$$

Simpson's 1/3 Rule

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n)+2(y_2+y_4+\cdots)+4[y_1+y_3+\cdots]\right]$$

$$= \frac{h}{3}\left[(y_0+y_n)+2(\text{Sum of even terms})+4[\text{Sum of odd terms}]\right]$$

Note:

Error in the trapezoidal rule is of the order h^2

Error in Simpson's 1/3 rule is of order h^4

. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by Trapezoidal rule, Simpson's $\frac{1}{3}$ rule and

Divide the interval (0,6) into 6 equal parts each of $h=1$.

The value of $f(x) = \frac{1}{1+x^2}$ are tabulated below.

x	0	1	2	3	4	5	6
$y=f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027

(i) By Trapezoidal rule,

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{2}\left[(y_0+y_6)+2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(1+0.027)+2(0.5+0.2+0.1+0.0588+0.0385)\right]$$

$$= 1.4108$$

(ii) By Simpson's 1/3 rule.

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.027)+4(0.5+0.1+0.0385)+2(0.2+0.0588)\right]$$

$$= 1.3662.$$

(5).

4. By dividing the range into ten equal parts, Evaluate $\int_0^{\pi} \sin x\, dx$ (or) by Trapezoidal and Simpson's rule. Verify your answer with Integration.

Soln:-

Range $= \pi - 0 = \pi$ Hence $h = \frac{\pi}{10}$.

We tabulate the values of y at different x's.

x	0	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	$6\pi/10$	$7\pi/10$	$8\pi/10$	$9\pi/10$	π
$y=\sin x$	0	0.3090	0.5878	0.8090	0.9511	1.0	0.9511	0.8090	0.5878	0.3090	0

(i) By Trapezoidal rule,

$$I = \frac{\pi}{20}\left[(0+0)+2(0.3090+0.5878+0.8090+0.9511+1.0+0.9511+0.8090+0.5878+0.3090)\right]$$

(ii) By Simpson's 1/3 rule,

$$I = \frac{h}{3}\left[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \left(\frac{\pi/10}{3}\right)\left[(0+0) + 4(0.3090+0.8090+1+0.8090+0.3090) + 2(0.5878+0.9511+0.9511+0.5878)\right]$$

$$= 2.0001.$$