SNS COLLEGE OF ENGINEERING

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Topic: 4.7 – Approximation of derivates using interpolation

Approximation of derivatives using interpolation polynomials

Forward Difference formula.

At $x = x_0$.

$$f'(x_0) = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 \cdots\right]$$

$$f''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \cdots\right]$$

$$f'''(x_0) = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right] \text{ and so on}$$

At $x \neq x_0$.

$$f'(x_0) = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \left(\frac{3u^2 - 6u + 2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3 - 18u^2 + 22u - 6}{24}\right)\Delta^4 y_0 + \cdots\right]$$

$$r = \frac{x_0 - x_0}{h}.$$

$$f''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \left(\frac{6u^2 - 18u + 11}{12}\right)\Delta^4 y_0 + \cdots\right]$$

$$f'''(x_0) = \Delta^3 y_0 + \frac{2u-3}{2} \Delta^4 y_0 + \cdots$$

Backward Difference Formula

At $x = x_0$

$$f'(x_0) = \frac{1}{3}\left[\nabla y_0 + \frac{1}{2} \nabla^2 y_0 + \frac{1}{3} \nabla^3 y_0 + \frac{1}{4} \nabla^4 y_0 - \cdots\right]$$

$$f''(x_0) = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12} \cdot \nabla^4 y_0 + \cdots\right]$$

$$f'''(x_0) = \frac{1}{3}\left[\nabla^3 y_0 + \frac{3}{2} \nabla^4 y_0 + \cdots\right]$$

At $x \neq x_0$. $\quad x + rh = x_0$

$$f'(x_0) = \nabla y_0 + \left(\frac{2u+1}{2}\right) \nabla^2 y_0 + \left(\frac{3u^2+6u+2}{6}\right) \nabla^3 y_0 + \frac{2u^3+9u^2+11u+3}{12} \nabla^4 y_0 + \cdots$$

$$u = \frac{x_0 - x}{h}$$

$$f''(x_0) = \nabla^2 y_0 + (u+1) \nabla^3 y_0 + \left[\frac{6u^2+18u+11}{12}\right] \nabla^4 y_0 + \cdots$$

$$f'''(x_0) = \nabla^3 y_0 + \frac{2r+3}{2} \cdot \nabla^4 y_0 + \cdots$$

③. Giventhat

x	1	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and y'' at $x=1.1$ and $x=1.6$

Soln:-

The difference table is formed as follows

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	7.989						
		0.414					
1.1	8.403		−0.036				
		0.378		0.006			
1.2	8.781		−0.03		−0.002		
		0.348		0.004		0.001	
1.3	9.129		−0.026		−0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		−0.023		0.002		
		0.299		0.005			
1.5	9.750		−0.018				
		0.281					
1.6	10.031						

To find $x = 1.1$

Using Newton's forward difference formula for differentiation,

$$\frac{dy}{dt} = \frac{1}{h}\left[\Delta y_0 + \left(\frac{2u-1}{2}\right)\Delta^2 y_0 + \left(\frac{3u^2-6u+2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3-18u^2+22u-6}{24}\right)\Delta^4 y_0 \dots\right.$$

$h = 0.1$

Here $u = \frac{1.1-1.0}{0.1} = 1$