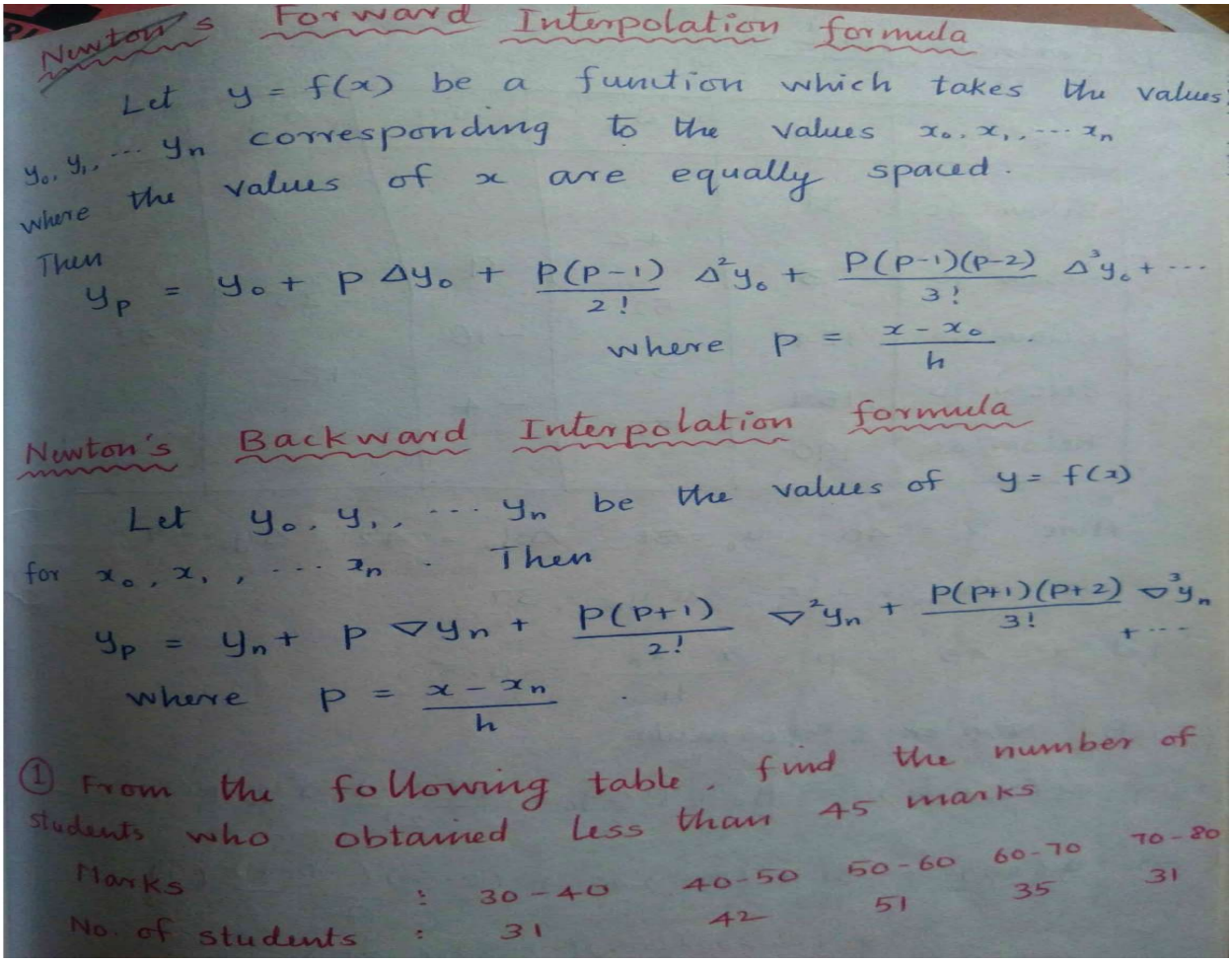
**Topic: 4.5 –** Newton's forward and backward interpolation



Newton's Forward Interpolation formula

Let $y = f(x)$ be a function which takes the values $y_0, y_1, \cdots y_n$ corresponding to the values $x_0, x_1, \cdots x_n$ where the values of $x$ are equally spaced.

Then

$$y_p = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

where $p = \frac{x - x_0}{h}$.

Newton's Backward Interpolation formula

Let $y_0, y_1, \cdots y_n$ be the values of $y = f(x)$ for $x_0, x_1, \cdots x_n$. Then

$$y_p = y_n + p\,\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

where $p = \frac{x - x_n}{h}$.

① From the following table, find the number of students who obtained less than 45 marks

| Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
|---|---|---|---|---|---|
| No. of students | 31 | 42 | 51 | 35 | 31 |

Difference table

| x | y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ | $\Delta^4 y$ |
|---|---|---|---|---|---|
| Below 40 | 31 | | | | |
| | | 42 | | | |
| Below 50 | 73 | | 9 | | |
| | | 51 | | −25 | |
| Below 60 | 124 | | −16 | | 37 |
| | | 35 | | 12 | |
| Below 70 | 159 | | −4 | | |
| | | 31 | | | |
| Below 80 | 190 | | | | |

Here $x_0 = 40$, $y_0 = 31$, $\Delta y_0 = 42$, $\Delta^2 y_0 = 9$, $\Delta^3 y_0 = -25$, $\Delta^4 y_0 = 37$

Let $x = 45$, $p = \frac{x - x_0}{h} = 0.5$

By Newton's formula

$$y(x_0 + ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots$$

$$\Rightarrow y(45) = 31 + (0.5)42 + \frac{(0.5)(-0.5)}{2}(9) + \frac{(0.5)(-0.5)(-1.5)}{6}(-25) + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{}(37)$$

$$= 31 + 21 - 1.125 - 1.5625 - 5.7813$$

$$y(45) = 43.5312$$

∴ No. of students less than 45 marks = 44

(2) Find the cubic polynomial which takes the value $y(0) = 1$, $y(1) = 0$, $y(2) = 1$ and $y(3) = 10$. Hence, find $y(4)$.

Difference table

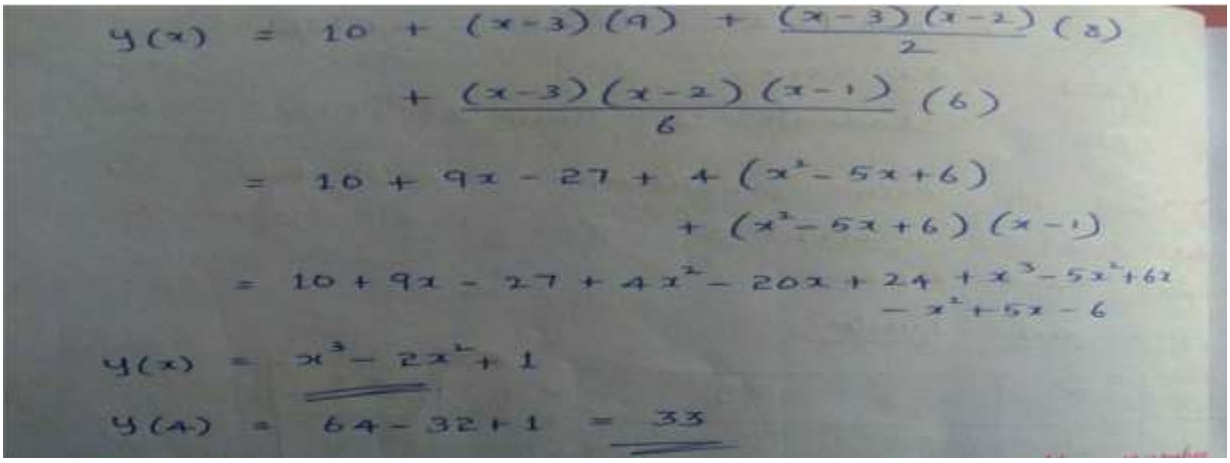
| x | y | $\nabla y_n$ | $\nabla^2 y$ | $\nabla^3 y$ |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | −1 | | |
| 1 | 0 | | 2 | |
| | | 1 | | 6 |
| 2 | 1 | | 8 | |
| | | 9 | | |
| 3 | 10 | | | |

Here $x_n = 3$, $\nabla y_n = 9$, $\nabla^2 y_n = 8$, $\nabla^3 y_n = 6$, $y_n = 10$

Let, Here $p = \frac{x - x_n}{h} = x - 3$

By Newton's formula

$$y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots$$


y(x) = 10 + (x-3)(9) + (x-3)(x-2)/2 (8)
+ (x-3)(x-2)(x-1)/6 (6)
= 10 + 9x - 27 + 4(x²-5x+6)
+ (x²-5x+6)(x-1)
= 10 + 9x - 27 + 4x² - 20x + 24 + x³ - 5x² + 6x
- x² + 5x - 6
y(x) = x³ - 2x² + 1
y(4) = 64 - 32 + 1 = 33