Topic: 4.1 – Introduction of application of Numerical differentiation and integration

Interpolation with unequal Intervals

Lagrange's Interpolation formula

Let $y_0, y_1, \cdots y_n$ be $(n+1)$ points of a function $y = f(x)$ where $f(x)$ is assumed to be a polynomial in x, corresponding to arguments $x_0, x_1, \cdots x_n$, not necessarily equally spaced.

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1$$

$$+ \cdots$$

$$+ \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} y_n$$

This is called the Lagrange's formula for interpolation.

Problems

1) Using Lagrange's interpolation formula, find the value of y corresponding to $x = 10$ from the following data

x :	5	6	9	11
y :	12	13	14	16

Given $x_0 = 5$, $x_1 = 6$, $x_2 = 9$, $x_3 = 11$

$y_0 = 12$, $y_1 = 13$, $y_2 = 14$, $y_3 = 16$

By Lagrange's interpolation formula.

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(10) = \frac{4 * 1 * (-1)}{(-1)(-4)(-6)} (12) + \frac{5(4)(-1)}{1(-3)(-5)} (13)$$

$$+ \frac{5(4)(-1)}{4(3)(-2)} (14) + \frac{5(4)(1)}{6(5)(2)} (16)$$

$$\therefore y(10) = 14.67$$

② Using Lagrange's interpolation formula, fit a polynomial to the following data

x	0	1	3	4
y	-12	0	6	12