Topic: 4.9 – Approximation of Derivatives

$$= \frac{1}{0.1}\left[0.414 + \left(\frac{2-1}{2}\right)(-0.036) + \left(\frac{3-6-2}{6}\right)(0.006) + \left(\frac{4-18+22-6}{24}\right)(-0.002) + \cdots\right]$$

$$= 3.9483.$$

To find $x = 1.6$

Using Newton's Backward difference formula for differentiation,

$$\frac{dy}{dt} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{1}{2}(-0.018) + \frac{1}{3}(0.005) + \frac{1}{4}(0.002) + \frac{1}{5}(0.003) + \frac{1}{6}(0.002)\right]$$

$$= 2.751$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.018 + 0.005 + \frac{11}{12}(0.002) + \cdots\right] = -1.1167$$

For $x = 1.1$,

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \left(\frac{6u^2-18u+11}{12}\right).\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.036 + (1-1)(0.006) + \left(\frac{6-18+11}{12}\right)(-0.002)\right]$$

$$= -3.5833.$$

(2) The table gives the distances in nautical miles of the visible horizon for the given heights in feet above the earth's surface.

X :	100	150	200	250	300	350	400
Y :	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find the values of y when x = 218 ft and 410 ft.

Solution.

The difference table is formed as follows.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63						
		2.4					
150	13.03		−0.39				
		2.01		0.15			
200	15.04		−0.24		−0.07		
		1.77		0.08		0.02	
250	16.81		−0.16		−0.05		0.02.
		1.61		0.03		0.04	
300	18.42		−0.13		−0.01		
		1.48		0.02			
350	19.90		−0.11				
		1.37					
400	21.27						

To find $x = 218$ ft, We use Newton's forward interpolation formula.

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \cdots$$

Where $u = \frac{218-100}{50} = 2.36.$

$$y(x) = 10.63 + \frac{(2.36)}{1}(2.4) + \frac{(2.36)(2.36-1)}{2}(-0.39) + \frac{(2.36)(2.36-1)(2.36-2)}{6}(0.15) + \frac{(2.36)(2.36-1)(2.36-2)(2.36-3)}{24}(0.07) + \cdots$$

$= 15.6948.$

To find $x = 410$ ft, We use Newton's Backward interpolation formula.

$$y(x) = y_6 + \frac{v}{1!}\nabla y_6 + \frac{v(v+1)}{2!}\nabla^2 y_6 + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_6 + \frac{v(v+1)(v+2)(v+3)}{4!}\nabla^4 y_6 + \cdots$$

Here $v = \frac{x - x_6}{h} = \frac{410-400}{50} = 0.2.$

$$\therefore y(x) = 21.27 + \frac{(0.2)}{1}(1.37) + \frac{(0.2)(0.2+1)}{2}(-0.11) + \frac{(0.2)(0.2+1)(0.2+2)}{6}(0.02) + \frac{(0.2)(0.2+1)(0.2+2)(0.2+3)}{24}(-0.01) + \cdots$$

$= 21.525.$