Topic: 4.6 – Problems on Newton's interpolation

③ From the data given below, find the number of students whose weight is given between 60 and 70

weight	0 – 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of students	250	120	100	70	50

Here $x_0 = 40$, $y_0 = 250$, $\Delta y_0 = 120$, $\Delta^2 y_0 = -20$

$\Delta^3 y_0 = -10$ and $\Delta^4 y_0 = 20$

Let $x = 70$, $p = \dfrac{x - x_0}{h} = \dfrac{70 - 40}{20} = 1.5$

By Newton's formula,

Difference Table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
Below 40	250				
		120			
Below 60	370		-20		
		100		-10	
Below 80	470		-30		20
		70		10	
Below 100	540		-20		
		50			
Below 120	590				

$y(x) = y_0 + p\,\Delta y_0 + \dfrac{P(P-1)}{2!}\Delta^2 y_0 + \cdots$

$y(70) = 250 + (1.5)(120) + \dfrac{(1.5)(0.5)}{2}(-20)$

$\qquad + \dfrac{(1.5)(0.5)(-0.5)}{6}(-10) + \dfrac{(1.5)(0.5)(-0.5)(-1.5)}{24}(20)$

$\qquad = 250 + 180 - 7.5 + 0.625 + 0.4688$

$y(70) = 423.5938$

No. of students less than 70 weight is 424

∴ No. of students between 60 and 70 weight is $424 - 370 = \underline{54}$

④ Population was recorded in a village as follows

Year   : 1941   1951   1961   1971   1981   1991
Population : 2500   2800   3200   3700   4350   5225

Estimate the population for the year 1945.

Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	2500					
		300				
1951	2800		100			
		400		0		
1961	3200		100		50	
		500		50		−25
1971	3700		150		25	
		650		75		
1981	4350		225			
		875				
1991	5225					

Here $x_0 = 1941$, $y_0 = 2500$, $\Delta y_0 = 300$, $\Delta^2 y_0 = 100$

$\Delta^3 y_0 = 0$, $\Delta^4 y_0 = 50$ and $\Delta^5 y_0 = -25$

Let $x = 1945$, $P = \dfrac{x - x_0}{h} = \dfrac{1945 - 1941}{10} = 0.4$

By Newton's formula,

$y(x) = y_0 + P\Delta y_0 + \dfrac{P(P-1)}{2!}\Delta^2 y_0 + \cdots$

$= 2500 + (0.4)(300) + \dfrac{(0.4)(-0.6)}{2}(100) + \dfrac{(0.4)(-0.6)(-1.6}{6}$

$+ \dfrac{(0.4)(-0.6)(-1.6)(-2.6)}{24}(50) + \dfrac{(0.4)(-0.6)(-1.6)(-2.6)(-3)}{120}(-25)$

$= 2500 + 120 - 12 - 2.08 - 0.7488$

$y(1945) = 2605.1712$

$\simeq \underline{2606}$

⑤ Using Newton's forward interpolation formula,
find the value of y when $x = 21$ from the following data.

x :   20   23   26   29
y : 0.3420   0.3907   0.4384   0.4848.

Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	0.3420			
		0.0487		
23	0.3907		−0.001	
		0.0477		−0.0003
26	0.4384		−0.0013	
		0.0464		
29	0.4848			