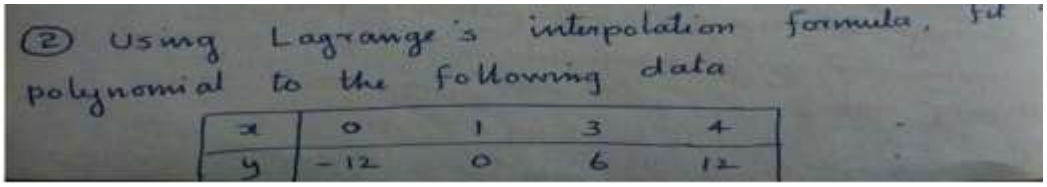
Topic: 4.2 –Lagrange's Interpolation



② Using Lagrange's interpolation formula, fit polynomial to the following data

x	0	1	3	4
y	-12	0	6	12

Given : $x_0 = 0$, $x_1 = 1$, $x_2 = 3$, $x_3 = 4$

$y_0 = -12$, $y_1 = 0$, $y_2 = 6$, $y_3 = 12$

By Lagrange's interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

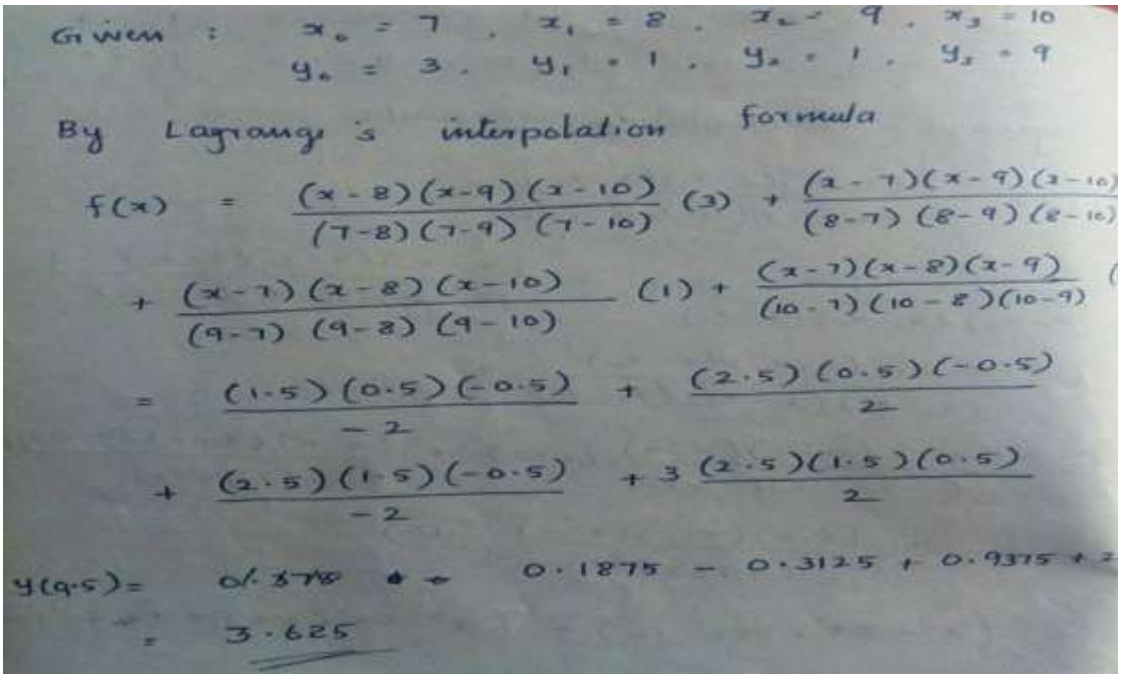
$$= \frac{(x-1)(x-3)(x-4)}{(-1)(-3)(-4)}(-12) + \frac{(x-0)(x-1)(x-4)}{(3)(2)(-1)}(6) + \frac{(x-0)(x-1)(x-3)}{(4)(3)(1)}(12)$$

$$= (x^3 - 8x^2 + 19x - 12) - (x^3 - 5x^2 + 4x) + x^3 - 4x^2 + 3x$$

$$f(x) = x^3 - 7x^2 + 18x - 12$$

③ Using Lagrange's formula of interpolation, find $y(9.5)$ given the data

x	7	8	9	10
y	3	1	1	9


Given : $x_0 = 7$, $x_1 = 8$, $x_2 = 9$, $x_3 = 10$
$y_0 = 3$, $y_1 = 1$, $y_2 = 1$, $y_3 = 9$
By Lagrange's interpolation formula
$f(x) = \frac{(x-8)(x-9)(x-10)}{(7-8)(7-9)(7-10)}(3) + \frac{(x-7)(x-9)(x-10)}{(8-7)(8-9)(8-10)}$
$+ \frac{(x-7)(x-8)(x-10)}{(9-7)(9-8)(9-10)}(1) + \frac{(x-7)(x-8)(x-9)}{(10-7)(10-8)(10-9)}$
$= \frac{(1.5)(0.5)(-0.5)}{-2} + \frac{(2.5)(0.5)(-0.5)}{2}$
$+ \frac{(2.5)(1.5)(-0.5)}{-2} + 3\frac{(2.5)(1.5)(0.5)}{2}$
$y(9.5) = 0.1875 - 0.3125 + 0.9375 +$
$= 3.625$