Topic: 3.10 – Eigen values of a matrix by power method.

1) find the largest eigen value and corresponding. eigen vector of the matrix given below using power method.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Let $x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$AX_0 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0+0 \\ -1+0+0 \\ 0+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

$$AX_1 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0.5+0 \\ -1-1+0 \\ 0+0.5+0 \end{bmatrix} = \begin{bmatrix} 2.5 \\ -2 \\ 0.5 \end{bmatrix} = 2.5 \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$\lambda X_2 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 0.8 + 0 \\ -1 - 1.6 - 0.2 \\ 0 + 0.8 + 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 2.8 \\ -2.8 \\ 1.2 \end{bmatrix} = 2.8 \begin{bmatrix} 1 \\ -1 \\ 0.428 \end{bmatrix} \quad 0.428$$

$$AX_3 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0.428 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 1 + 0 \\ -1 - 2 - 0.488 \\ 0 + 1 + 0.856 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -3.488 \\ 1.856 \end{bmatrix} = 3.488 \begin{bmatrix} 0.875 \\ -1 \\ 0.541 \end{bmatrix}$$

$$AY_4 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.875 \\ -1 \\ 0.541 \end{bmatrix}$$

$$= \begin{bmatrix} 1.75 + 1 + 0 \\ -0.875 - 2 - 0.541 \\ 0 + 1 + 1.082 \end{bmatrix} = \begin{bmatrix} 2.74 \\ 3.41 \\ 2.08 \end{bmatrix} = 3.41 \begin{bmatrix} 0.80 \\ 1 \\ 0.61 \end{bmatrix}$$

$$AX_5 = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.805 \\ -1 \\ 0.609 \end{bmatrix}$$

$$= \begin{bmatrix} 1.61 + 1 \\ -0.805 - 2 - 0.609 \\ 0 + 1 + 1.218 \end{bmatrix}$$

$$= \begin{bmatrix} 2.61 \\ -3.414 \\ 2.218 \end{bmatrix} = 3.414 \begin{bmatrix} 0.764 \\ -1 \\ 0.650 \end{bmatrix}$$

$$A \times b = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.764 \\ -1 \\ 0.650 \end{bmatrix}$$

$$= \begin{bmatrix} 1.528 + 1 + 0 \\ -0.764 - 2 - 0.650 \\ 0 + 1 + 1.3 \end{bmatrix}$$

$$= \begin{bmatrix} 2.528 \\ -3.414 \\ 2.3 \end{bmatrix}$$

$$= 3.414 \begin{bmatrix} 2.528 \\ -1 \\ 2 \end{bmatrix} = 3.414 \begin{bmatrix} 0.740 \\ 1 \\ 0.674 \end{bmatrix}$$

eigen value $\lambda = 3.414$

eigen vector $x = (0.740, 1, 0.674)$