Topic: 3.3 – Pivoting Gauss Jordan method

Gauss - Jordan Method

(2) Apply Gauss - Jordan method to find the solution of the following system:

$10x + y + z = 12$; $2x + 10y + z = 13$;

$x + y + 5z = 7$.

Since the coefficient of x in the last equation is unity, we rewrite the equations interchanging the first and the last. Hence the augmented matrix is

$$(A, B) = \left(\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{array}\right)$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 10R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 1 & -\frac{9}{8} & -1/8 \\ 0 & -9 & -49 & -58 \end{array}\right] R_2 \to R_2/8$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 1 & -9/8 & -1/8 \\ 0 & 0 & -\frac{473}{8} & -\frac{473}{8} \end{array}\right] \quad R_3 \to R_3 + 9R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & \frac{49}{8} & \frac{57}{8} \\ 0 & 1 & -9/8 & -1/8 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ \\ R_3 \to \left(-\frac{8}{473}\right) R_3 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - \frac{49}{8} R_3 \\ R_2 \to R_2 + \frac{9}{8} R_3 \end{array}$$

$\therefore \quad x = 1, \quad y = 1, \quad z = 1$

③ Solve the following system by Gauss - Jordan method : $3x - y + 2z = 12$, $x + 2y + 3z = 11$, $2x - 2y - z = 2$.

Since the coff. of x in the second equation is unity, we rewrite the equations interchanging the first and the second. Hence the augmented matrix is

$$(A, B) = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 3 & -1 & 2 & 12 \\ 2 & -2 & -1 & 2 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & -7 & -7 & -21 \\ 0 & -6 & -7 & -20 \end{array}\right] \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 11 \\ 0 & 1 & 1 & 3 \\ 0 & -6 & -7 & -20 \end{array}\right] R_2 \to \frac{R_2}{-7}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & -2 \end{array}\right] \begin{array}{l} R_1 \to R_1 - 2R_2 \\ \\ R_3 \to R_3 + 6R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right] R_3 \to -R_3$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - R_3 \end{array}$$

$\therefore \; x = 3 \; ; \; y = 1 \; ; \; z = 2$

4) Solve by Gauss-Jordan method:

$5x_1 - x_2 = 9 \; ; \; -x_1 + 5x_2 - x_3 = 4 \; ; \; -x_2 + 5x_3 = -6$