Topic: 3.9 – Matrix Inversion – Gauss Jordan method

Inverse of a Matrix by Gauss - Jordan

① Find the inverse of $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ by Gauss - Jordan method.

consider.

$$(A/I) = \left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \quad R_1 \leftrightarrow R_2$$

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$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & -5 & -8 & 0 & -3 & 1 \end{array}\right] \quad R_3 \rightarrow R_3 - 3R_1$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 & -3 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 + 5R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & -2 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5/2 & -3/2 & 1/2 \end{array}\right] R_3 \to R_3/2$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 1 & 5/2 & -3/2 & 1/2 \end{array}\right] \begin{array}{l} R_1 \to R_1 + R_3 \\ R_2 \to R_2 - 2R_3 \end{array}$$

Hence $A^{-1} = \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{pmatrix}$

(2) By Gauss - Jordan method, find A^{-1} if

$A = \begin{pmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{pmatrix}$

consider $(A/I) = \left[\begin{array}{ccc|ccc} 4 & 1 & 2 & 1 & 0 & 0 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 1 & -2 & 2 & 0 & 0 & 1 \end{array}\right]$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 0 & 0 & 1 \\ 2 & 3 & -1 & 0 & 1 & 0 \\ 4 & 1 & 2 & 1 & 0 & 0 \end{array}\right] \quad R_1 \leftrightarrow R_3$$

$-9 + \frac{45}{7}$

$-4 + \frac{12}{7}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 0 & 0 & 1 \\ 0 & 7 & -5 & 0 & 1 & -2 \\ 0 & 9 & -6 & 1 & 0 & -4 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & -2 & 2 & 0 & 0 & 1 \\ 0 & 1 & -5/7 & 0 & 1/7 & -2/7 \\ 0 & 9 & -6 & 1 & 0 & -4 \end{array}\right] \quad R_2 \to R_2/7$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 4/7 & 0 & 2/7 & -3/7 \\ 0 & 1 & -5/7 & 0 & 1/7 & -2/7 \\ 0 & 0 & -18/7 & 1 & -9/7 & -\frac{10}{7} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + 2R_2 \\ \\ R_3 \to R_3 - 9R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 4/7 & 0 & 2/7 & -3/7 \\ 0 & 1 & -5/7 & 0 & 1/7 & -2/7 \\ 0 & 0 & 1 & -\frac{7}{18} & \frac{1}{2} & \frac{5}{9} \end{array}\right] \quad R_3 \to -\frac{7}{18} R_3$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/9 & 0 & -1/9 \\ 0 & 1 & 0 & -5/18 & 1/2 & 1/9 \\ 0 & 0 & 1 & -7/18 & 1/2 & 5/9 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - \frac{4}{7} R_3 \\ R_2 \to R_2 + \frac{5}{7} R_3 \end{array}$$

(3) By Gauss-Jordan method, find A^{-1} if

$$A = \begin{pmatrix} 2 & 2 & 6 \\ 2 & 6 & -6 \\ 4 & -8 & 8 \end{pmatrix}.$$

Consider

$$(A/I) = \left[\begin{array}{ccc|ccc} 2 & 2 & 6 & 1 & 0 & 0 \\ 2 & 6 & -6 & 0 & 1 & 0 \\ 4 & -8 & 8 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1/2 & 0 & 0 \\ 2 & 6 & -6 & 0 & 1 & 0 \\ 4 & -8 & 8 & 0 & 0 & 1 \end{array}\right] \quad R_1 \to R_1/2$$

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$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 4 & -12 & -1 & 1 & 0 \\ 0 & -12 & -4 & -2 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 4R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & -3 & -\frac{1}{4} & \frac{1}{4} & 0 \\ 0 & -12 & -4 & -2 & 0 & 1 \end{array}\right] \quad R_2 \to R_2/4$$

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$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & [illegible] & [illegible] & [illegible] & 0 \\ 0 & 1 & -3 & -\frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & -40 & -5 & 3 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_3 \to R_3 + 12R_2 \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{4} & -\frac{1}{4} & 0 \\ 0 & 1 & -3 & -\frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & \frac{1}{8} & -\frac{3}{40} & -\frac{1}{40} \end{array}\right] \quad R_3 \to -\frac{R_3}{40}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{5}\ \frac{8}{40} & \frac{3}{20} \\ 0 & 1 & 0 & \frac{1}{8} & \frac{1}{40} & -\frac{3}{40} \\ 0 & 0 & 1 & \frac{1}{8} & -\frac{3}{40} & -\frac{1}{40} \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 6R_3 \\ R_2 \to R_2 + 3R_3 \end{array}$$

$-\frac{10}{4} + \frac{18}{40}$