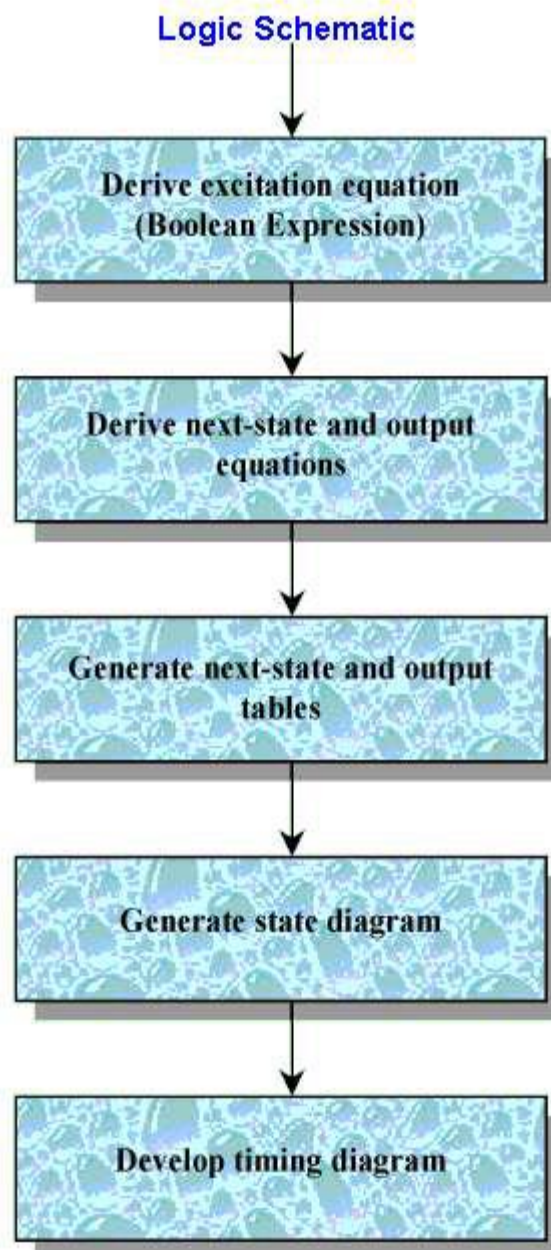


Analysis of Sequential Circuits

The behaviour of a sequential circuit is determined from the inputs, the outputs and the states of its flip-flops. Both the output and the next state are a function of the inputs and the present state. The suggested analysis procedure of a sequential circuit is set out.

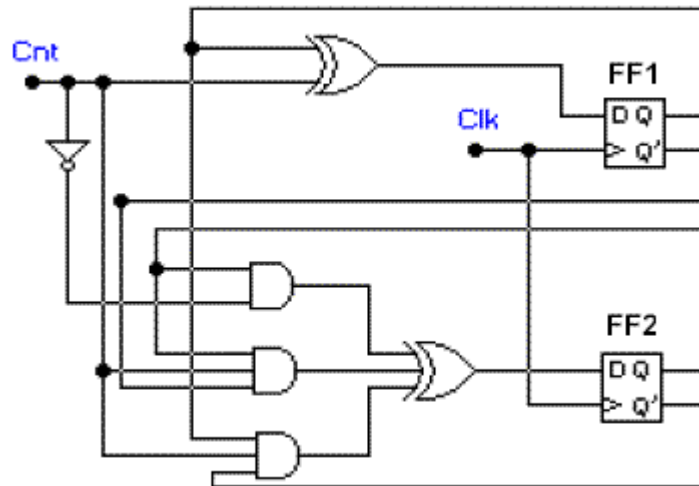


Analysis procedure of sequential circuits.

It is started with the logic schematic from which we can derive excitation equations for each flip-flop input. Then, to obtain next-state equations, insert the excitation equations into the characteristic equations. The output equations can be derived from the schematic, and once we have our output and next-state equations, can generate the

next-state and output tables as well as state diagrams. When we reach this stage, we use either the table or the state diagram to develop a timing diagram which can be verified through simulation.

Derive the state table and state diagram for the sequential circuit.



Logic schematic of a sequential circuit.

SOLUTION:

STEP 1: First we derive the Boolean expressions for the inputs of each flip-flops in the schematic, in terms of external input Cnt and the flip-flop outputs Q1 and Q0. Since there are two D flip-flops in this example, we derive two expressions for D1 and D0:

$$D0 = Cnt Q0 = Cnt' * Q0 + Cnt * Q0'$$

$$D1 = Cnt' * Q1 + Cnt * Q1' * Q0 + Cnt * Q1 * Q0'$$

These Boolean expressions are called **excitation equations** since they represent the inputs to the flip-flops of the sequential circuit in the next clock cycle.

STEP 2: Derive the next-state equations by converting these excitation equations into flip-flop characteristic equations. In the case of D flip-flops, Q(next) = D. Therefore the next state equal the excitation equations.

$$Q0(next) = D0 = Cnt' * Q0 + Cnt * Q0'$$

$$Q1(next) = D1 = Cnt' * Q1 + Cnt * Q1' * Q0 + Cnt * Q1 * Q0'$$

STEP 3: Now convert these next-state equations into tabular form called the next-state table.

Present State Q1Q0	Next State	
	Cnt = 0	Cnt = 1
00	00	01
01	01	10
10	10	11
11	11	00

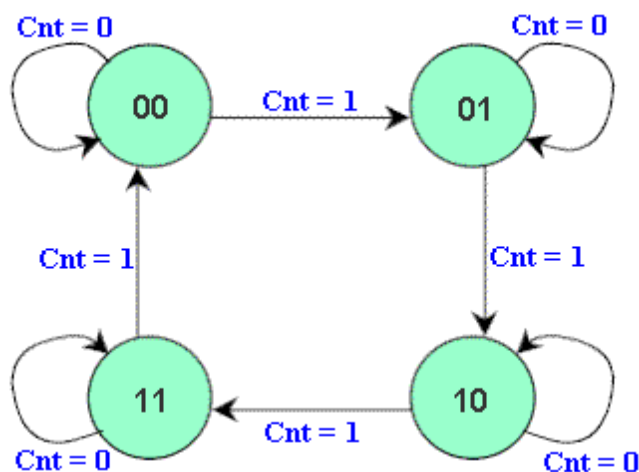
Each row is corresponding to a state of the sequential circuit and each column represents one set of input values. Since have two flip-flops, the number of possible states is four - that is, Q1Q0 can be equal to 00, 01, 10, or 11. These are present states as shown in the table.

For the next state part of the table, each entry defines the value of the sequential circuit in the next clock cycle after the rising edge of the **Clk**. Since this value depends on the present state and the value of the input signals, the next state table will contain one column for each assignment of binary values to the input signals. In this example, since there is only one input signal, **Cnt**, the next-state table shown has only two columns, corresponding to **Cnt = 0** and **Cnt = 1**.

Note that each entry in the next-state table indicates the values of the flip-flops in the next state if their value in the present state is in the row header and the input values in the column header.

Each of these next-state values has been computed from the next-state equations in STEP 2.

STEP 4: The state diagram is generated directly from the next-state table.



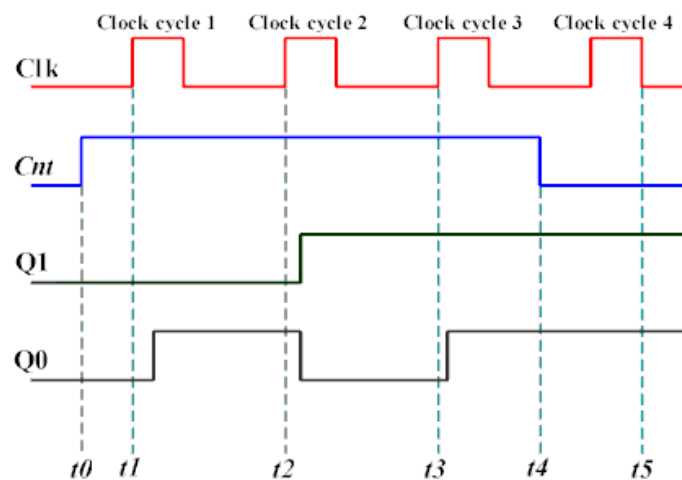
Each arc is labelled with the values of the input signals that cause the transition from the present state (the source of the arc) to the next state (the destination of the arc).

In general, the number of states in a next-state table or a state diagram will equal 2^m , where m is the number of flip-flops. Similarly, the number of arcs will equal $2^m \times 2^k$, where k is the number of binary input signals. Therefore, in the state diagram, there must be four states and eight transitions. Following these transition arcs, that as long as $Cnt = 1$, the sequential circuit goes through the states in the following sequence:

0, 1, 2, 3, 0, 1, 2,.... On the other hand, when $Cnt = 0$, the circuit stays in its present state until Cnt changes to 1, at which the counting continues.

Since this sequence is characteristic of modulo-4 counting, it can conclude that the sequential circuit is a modulo-4 counter with one control signal, Cnt , which enables counting when $Cnt = 1$ and disables it when $Cnt = 0$.

Below, we show a timing diagram, representing four clock cycles, which enables to observe the behaviour of the counter in greater detail.



Timing Diagram

In this timing diagram assumed that Cnt is asserted in clock cycle 0 at t_0 and is disasserted in clock cycle 3 at time t_4 . We have also assumed that the counter is in state $Q_1Q_0 = 00$ in the clock cycle 0. Note that on the clock's rising edge, at t_1 , the counter will go to state $Q_1Q_0 = 01$ with a slight propagation delay; in cycle 2, after t_2 , to $Q_1Q_0 = 10$; and in cycle 3, after t_3 to $Q_1Q_0 = 11$. Since Cnt becomes 0 at t_4 , know that the counter will stay in state $Q_1Q_0 = 11$ in the next clock cycle.