Travelling Salesperson Problem (TSP):-

Row minimization

$$Red\_Row(M) = \left[M_{ij} - \min\{ M_{ij} \mid 1 \le j \le n \} \right]$$

$$M = \begin{bmatrix} \infty & 20 & 30 & 10 & 11 \\ 15 & \infty & 16 & 4 & 2 \\ 3 & 5 & \infty & 2 & 4 \\ 19 & 6 & 18 & \infty & 3 \\ 16 & 4 & 7 & 16 & \infty \end{bmatrix} \begin{matrix} 10 \\ 2 \\ 2 \\ 3 \\ 4 \\ \hline 21 \end{matrix}$$

$$Red\_Row(M) = \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

Column minimization

$$Red\_col(M) = M_{ji} - \min\{ M_{ji} \mid 1 \le j \le n \} \text{ where } M_{ji}$$

$$M = \begin{bmatrix} \infty & 10 & 20 & 0 & 1 \\ 13 & \infty & 14 & 2 & 0 \\ 1 & 3 & \infty & 0 & 2 \\ 16 & 3 & 15 & \infty & 0 \\ 12 & 0 & 3 & 12 & \infty \end{bmatrix}$$

$$1 \quad \underbrace{\uparrow \quad \uparrow \quad \uparrow \quad \uparrow}_{\text{Ignore}} = 4$$

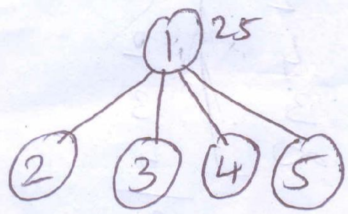
$$Red\_column(M) = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 14 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

Total cost = cost (Red_Row(M)) + cost (Red_column(M))

21 + 4

$$M = \begin{bmatrix} \infty & 10 & 17 & 0 & 1 \\ 12 & \infty & 11 & 2 & 0 \\ 0 & 3 & \infty & 0 & 2 \\ 15 & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & 12 & \infty \end{bmatrix}$$

optimum cost = 21 + 4 = 25



consider path 1, 2 make 1st row & 2nd column as ∞. & set M[2][1] = ∞

$$\underbrace{\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 11 & 2 & 0 \\ 0 & \infty & \infty & 0 & 2 \\ 15 & \infty & 12 & \infty & 0 \\ 11 & \infty & 0 & 12 & \infty \end{bmatrix}}_{\text{Ignore}} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \Bigg\} \text{ Ignore}$$

Cost of node 2 is

optimum cost + reduced cost + old value of M[1][2]

$25 + 0 + \underbrace{10}_{\text{Ignore}}$

= 35

consider path 1, 3 make 1st row & 3rd column as ∞ & set M[3][1] = ∞

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & 2 & 0 \\ \infty & 3 & \infty & 0 & 2 \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \Bigg\} \text{ Ignore}$$

Cost of node 3 is

25 + 11 + 17

consider path 1, 4 make 1st row & 4th column as ∞ & set
$M[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & 0 \\ 0 & 3 & \infty & \infty & 2 \\ \infty & 3 & 12 & \infty & 0 \\ 11 & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \Big\} \text{Ignore}$$

$\downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow$ Ignore

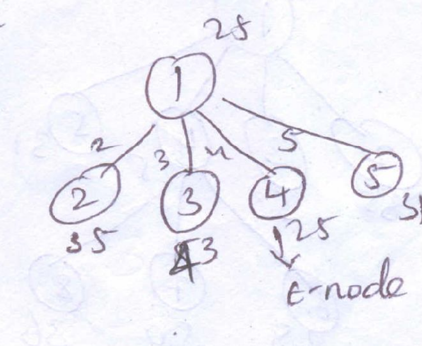
cost of node 4 is $= 25 + 0 + 0 = 25$

consider path 1, 5 make 1st row & 5th column as ∞ & set
$M[5][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & 2 & \infty \\ 0 & 3 & \infty & 0 & \infty \\ 15 & 3 & 12 & \infty & \infty \\ \infty & 0 & 0 & 12 & \infty \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow 2 \\ \rightarrow \\ \rightarrow 3 \\ \rightarrow \end{matrix} \Big\} \text{Ignore}$$

$\downarrow \; \downarrow \; \downarrow \; \downarrow \; \downarrow$ ignore

cost of node 5 is
$25 + 5 + 1 = 31$



now as cost of node 4 is optimum, we will set node 4 as E-node & generate its children nodes 6, 7, 8

consider path 1, 4, 2 for node 6. set 1st row, 4th row & 2nd & 4th column as ∞ & $M[4][1] = \infty$ $M[2][1] = \infty$

Consider path 1,4,3 for node 7. set 1st row, 4th row & as ∞. set 4th column, 3rd column as ∞. set $M[4][1] = \infty$
$M[3][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & \infty & \infty & 0 \\ \infty & 3 & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & 0 & \infty & \infty & \infty \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \rightarrow 2 \\ \rightarrow \\ \rightarrow \end{matrix} \text{ Ignore}$$

$$\begin{matrix} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 11 & \underbrace{\quad\quad\quad\quad\quad\quad}_{\text{Ignore}} \end{matrix}$$

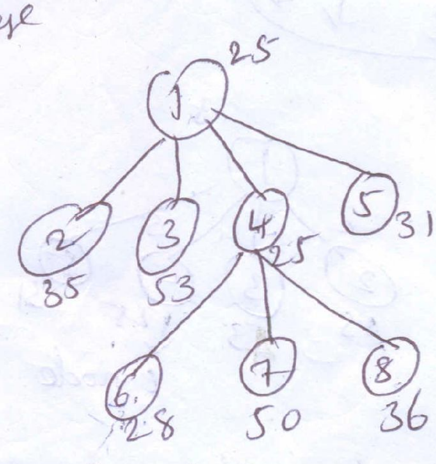
cost of node 7 is
$25 + 13 + 12 = 50$

Consider path 1,4,5 for node 8. set 1st & 4th row as ∞
set 4th column & 5th column as ∞. set $M[5][1] = \infty$
$[4][1] = \infty$

$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 12 & \infty & 11 & \infty & \infty \\ 0 & 3 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & 0 & 0 & \infty & \infty \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow 11 \\ \rightarrow \\ \rightarrow \\ \rightarrow \end{matrix} \text{ Ignore}$$

$$\underbrace{\begin{matrix} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \end{matrix}}_{\text{Ignore}}$$

cost of node 8 is
$25 + 11 + 0 = 36$



Now as cost of node 6 is min, node 6 become an E-node. hence generate children for node 6

node 9 & 10 are children node of 6.

Consider path 1,4,2,3 for node 9. set 1st row

$$\begin{array}{c|c}
\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & 2 \\ \infty & \infty & \infty & \infty & \infty \\ 11 & \infty & \infty & \infty & \infty \end{bmatrix} & \begin{array}{l} \\ \\ 2 \\ \text{Ignore} \\ 11 \end{array} \\ \hline
\begin{array}{ccccc} 11 & & & & 2 \end{array} &
\end{array}$$

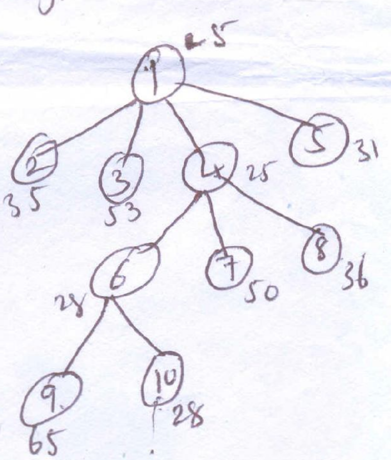
Cost of node 9 is
$= 28 + 26 + 11$
$= 65$

consider path 1, 4, 2, 5 for node 10. set 1st, 4th & 2nd row as ∞. set 4th, 2nd & 5th column as ∞. set $M[4][1] = \infty$, $[2][1] = \infty$, $[5][1] = \infty$

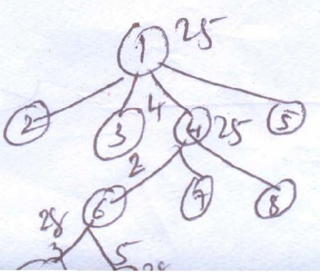
$$\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & \infty & \infty \end{bmatrix} \text{ Ignore}$$

Ignore

cost of node 10 is
$= 28 + 0 + 0 = 28$



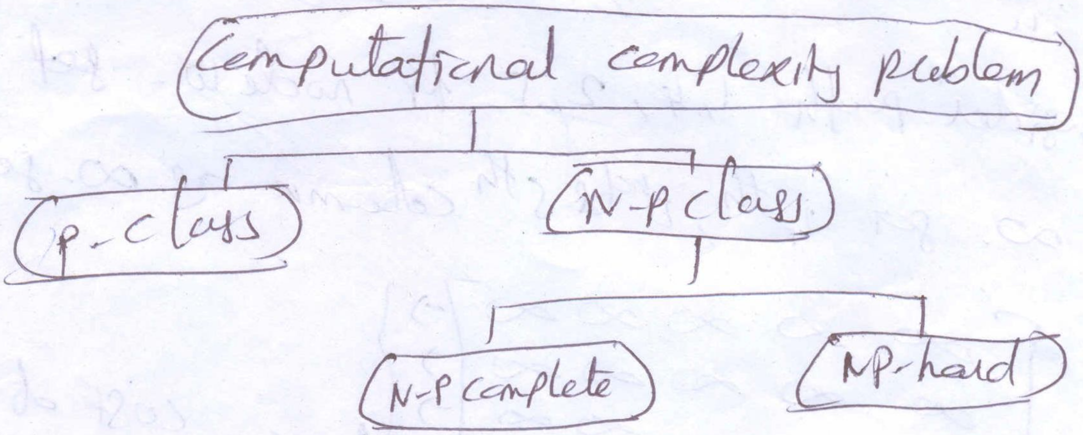
node 10 becomes E-node now
As for node 10 only child being generated is node 11. we get path as 1, 4, 2, 5, 3. to complete the tour we return to 1. hence the state space tree is



NP Hard & NP complete problems - Basic concepts

The classes P & NP-

-> P stands for polynomial which solved polynomial time
-> NP stands for non-deterministic polynomial time.



Proving NP problems

-> hamiltonian cycle
-> CIRCUIT - SAT

Satisfyiability problem

-> CNF - SAT problem
-> 3 SAT problem

Properties:-

$P \subseteq NP$

$P = NP$

$P \neq NP$

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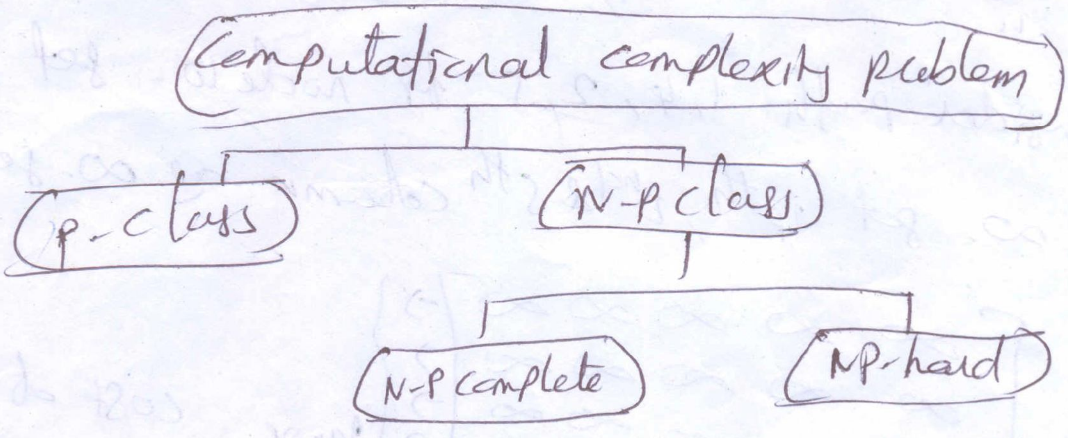

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