Topic: 3.1 – Introduction - Newton Raphson method

Newton - Raphson Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \ldots$$

Convergence criterion

Let $g(x) = x - \dfrac{f(x)}{f'(x)}$

$$|g'(x)| < 1 \Rightarrow |f(x) f''(x)| < [f'(x)]^2$$

is the desired condition for convergence.

Order of convergence

If the root is simple, then the convergence of Newton - Raphson method is quadratic.

Problems

(1) Find the positive root of $x^4 - x - 10 = 0$ by Newton's method, correct to 4 decimal places.

Let $f(x) = x^4 - x - 10$

$f'(x) = 4x^3 - 1$

Newton's formula,

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$= x_i - \frac{x_i^4 - x_i - 10}{4x_i^3 - 1}$$

$$x_{i+1} = \frac{3x_i^4 + 10}{4x_i^3 - 1}, \quad i = 0, 1, 2, \ldots$$

Since $f(0) = -10 < 0$

$f(1) = -10 < 0$

$f(2) = 5 > 0$, the root lies between 1 and 2.

We choose $x_0 = 1.5$.

$$x_1 = \frac{3(1.5)^4 + 10}{4(1.5)^3 - 1} = \frac{25.1875}{12.5} = 2.015$$

$x_2 = 1.8741$; $x_3 = 1.8558$

$x_4 = 1.8556 \quad ; \quad x_5 = 1.8556$

Hence the root is $\underline{1.8556}$

(2) Find by Newton's method, the real of $x \log_{10} x = 1.2$, correct to 4 decimal p

Let $f(x) = x \log_{10} x - 1.2$

$$f'(x) = \log_{10} x + x \frac{d}{dx} [\log_{10} x]$$

$$= \log_{10} x + x \frac{d}{dx} [\log_e x \cdot \log_{10} e$$

$$f'(x) = \log_{10} x + x \left(\frac{1}{x}\right) \log_{10} e$$

$$f'(x) = \log_{10} x + \log_{10} e$$

$$f'(x) = \log_{10} x + 0.4343$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, \quad i = 0, 1, 2, \ldots$$

Since $f(0) < 0$, $f(1) < 0$, $f(2) < 0$

$f(3) > 0$

$\therefore$ The root lies between 2 and 3.

We choose $x_0 = 2.5$

$$x_1 = 2.5 - \frac{2.5 \log_{10} 2.5 - 1.2}{\log_{10} 2.5 + 0.4343}$$

$$= 2.5 - \left[\frac{-0.2051}{0.8322}\right] = 2.7465$$

$x_2 = 2.7406$; $x_3 = 2.7406$

Hence the root is 2.7406