IAE I ANSWER KEY

PART-A.

1. $f(x) = K(1+x)$

$K = 2/27$ $\quad k = 2/27.$

2. $P[X=x] = \frac{e^{-\lambda}\lambda^x}{x!}$;

Mean $= E[X] = \lambda$

variance : $E[X^2] - E[X]^2 = \lambda$.

3. If the random variable X follows exponential distribution, then $P[X > s+t / X > t] = P[X > s]$ for all $s, t > 0$.

4. $f(x,y) = K e^{-(2x+3y)}$

$K = 6$ ..

5. $f(x,y) = 8xy, \quad 0 < x < y < 1$.

$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = 4x^3$.

PART-B.

6.a) i) $K = 0.067$

ii) $P(X<2) = P(X=-2)+P(X=-1)+P(X=0)+P(X$

$+P(X$

$P(X<2) = 0.501$

$P(-2<X\le 2) = P(X=-1)+P(X=0)+P(X=1)+P(X=2$

$P(-2<X\le 2) = 0.701$

iii)

x	$P(X\le x)$
-2	0.1
-1	0.167
0	0.367
1	0.501
2	0.801
3	1

iv) $E[X]=\Sigma x\, p(x)$

$E[X] = 0.502$

6 b) $P[X=x] = nC_x p^x q^{n-x}$.

MGF; $M_X(t) = (pe^t - q)^n$

$\text{Mean} = np$

variance: $E[X^2]-E[X]^2$

$=(n^2p^2+npq)-(n^2p^2)$

$\text{var}(X) = npq$

7.
a) (i) $\lambda = 1.8$; $P[X=x] = \frac{e^{-\lambda}\lambda^{x}}{x!}$

(i) Without breakdown:

$P[X=0] = 0.1653$

ii) With only one breakdown:

$P[X=1] = 0.2975$

iii) With atleast one breakdown:

$P[X \geq 1] = 1 - P[X < 1]$

$= 0.8347.$

$\therefore P[X<1] = 0.1653.$

7 a) ii) $f(x) = \lambda e^{-\lambda x}, \; x \geq 0$

$0 \quad , x < 0$

MGF : $M_X(t) = \frac{\lambda}{\lambda - t}$

Mean $= 1/\lambda$

Variance : $E[X^2] - E[X]^2 = \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$

$Var(x) = 1/\lambda^2.$

7 b) i) $f(x,y) = c\,x(x-y),\ 0 < x < 2,\ -x < y < x.$

(i) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1.$

$\therefore C = 1/8.$

ii) Marginal PDF's

$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{-x}^{x} \frac{1}{8}(x^2 - xy)\,dy = \frac{x^3}{4}.$

$f(y) = \frac{1}{12}(4-3y).$

iii) conditional density y/x.

$f(y/x) = \frac{f(x,y)}{f(x)} = \frac{x-y}{2x^2}.$

PART - C.

8 a)

$x \backslash y$	1	2	3	$P(X=x)$
0	3K	6K	9K	18K
1	5K	8K	11K	24K
2	7K	10K	13K	30K
$P(Y=y)$	15K	24K	33K	72K.

$$\sum_j \sum_i P(x_i, y_j) = 1$$

$$K = 1/72.$$

conditional distribution of X Gn Y=y.

$Y=1 \Rightarrow P[X=0/Y=1] = 1/5$

$P[X=1/Y=1] = 1/3$

$P[X=2/Y=1] = 7/15.$

$Y=2 \Rightarrow P[X=0/Y=2] = 1/4$

$P[X=1/Y=2] = 1/3$

$P[X=2/Y=2] = 5/12$

$Y=3 \Rightarrow P[X=0/Y=3] = 3/11$

$P[X=1/Y=3] = 1/3$

$P[X=2/Y=3] = 13/33.$

conditional distribution of Y given X=x

$x=0 \Rightarrow P[Y=1/X=0] = 1/6$

$P[Y=2/X=0] = 1/3$

$P[Y=3/X=0] = 1/2$

$x=1 \Rightarrow P[Y=1/X=1] = 5/24$

$P[Y=2/X=1] = 1/3$

$P[Y=3/X=1] = 11/24$

$x=2 \Rightarrow P[Y=1/X=2] = 7/30$

$P[Y=2/X=2] = 1/3$

$P[Y=3/X=2] = 13/30$

X+Y	P
1: $P(0,1)$	3/72
2: $P(0,2)+P(1,1)$	11/72
3: $P(0,3)+P(1,2)+P(2,1)$	24/72
4: $P(1,3)+P(2,2)$	21/72
5	13/72

8 b) i) $n=4;\ p=1/2;\ q=1/2$

i) $P[X=x] = nC_x\, p^x\, q^{n-x}$.

$P[X=2] = 3/8 \Rightarrow 800 \times \frac{3}{8} = 300$

ii) $P[X \geq 1] = 1 - P[X<1] = 1 - P[X=0]$

$= 15/16 \Rightarrow 800 \times 15/16 = 750$

iii) $P[X \geq 2] = 1 - P[X<2] = 1 - [P(X=0)+P(X=1)]$

$= 11/16 \Rightarrow 800 \times 11/16 = 550$

iv) $1 - \{P[\text{all are boys}] + P[\text{all are girls}]\}$

$= 1 - \{P[X=4] + P[X=0]\} = 7/8 \Rightarrow 800 \times \frac{7}{8} = 700$

8b)

(ii) $P[X=x] = \frac{e^{-\lambda}\lambda^x}{x!}$

MGF; $M_X(t) = e^{\lambda(e^t-1)}$

Mean: $E[X] = \lambda$

variance: $E[X^2] - E[X]^2 = (\lambda^2+\lambda) - \lambda^2$

$\text{var}(X) = \lambda$