



TOPIC 2.10- Linear Regression

1)

The equation of two regression lines are

$3\bar{x} + 12\bar{y} = 19$ and $3\bar{y} + 9\bar{x} = 46$. Find \bar{x} , \bar{y}
and the correlation coefficient between x and y

Since both the regression passing through
 (\bar{x}, \bar{y}) , we get

$$3\bar{x} + 12\bar{y} = 19 \rightarrow \textcircled{1}$$

$$9\bar{x} + 3\bar{y} = 46 \rightarrow \textcircled{2}$$

$$\textcircled{1} \times 3 \quad \begin{array}{r} 9\bar{x} + 36\bar{y} = 57 \\ \underline{- 9\bar{x} + 3\bar{y} = 46} \\ -33\bar{y} = -11 \end{array}$$

$$-33\bar{y} = -11$$

$$\boxed{\bar{y} = \frac{1}{3}}$$

$$3\bar{x} + 12\bar{y} = 19$$

$$3\bar{x} + 4 = 19$$

$$3\bar{x} = 15$$

$$\boxed{\bar{x} = 5}$$



$$\begin{aligned} \textcircled{1} \Rightarrow 3x + 12y &= 19 & \textcircled{2} \Rightarrow 9x + 3y &= 46 \\ 12y &= -3x + 19 & 9x &= -3y + 46 \\ y &= -\frac{1}{4}x + \frac{19}{12} & x &= -\frac{1}{3}y + \frac{46}{9} \\ \Rightarrow \boxed{b_{yx} = -\frac{1}{4}} & & \Rightarrow \boxed{b_{xy} = -\frac{1}{3}} & \end{aligned}$$

The correlation coefficient

$$r = \pm \sqrt{b_{xy} b_{yx}} = \pm \sqrt{\left(-\frac{1}{3}\right)\left(-\frac{1}{4}\right)}$$

$$r = \pm \sqrt{\frac{1}{12}} = -\frac{1}{2\sqrt{3}}$$

2) The tangent of the angle between the lines of regression of y on x and x on y is 0.6 and $\sigma_x = \frac{1}{2}\sigma_y$, find the correlation between x and y .

$$\tan \alpha = \frac{1-r^2}{r} \left(\frac{\sigma_x \sigma_y}{\sigma_x^2 + \sigma_y^2} \right)$$

$$0.6 = \frac{1-r^2}{r} \left(\frac{\frac{1}{2}\sigma_y^2}{\left(\frac{1}{2}\sigma_y\right)^2 + \sigma_y^2} \right)$$

$$\frac{3}{5} = \frac{1-r^2}{r} \left[\frac{\frac{1}{2}}{\frac{1}{4} + 1} \right]$$



$$\Rightarrow \frac{3}{5} = \frac{1-\gamma^2}{\gamma} \left[\frac{1/2}{5/4} \right]$$

$$\Rightarrow \frac{3}{5} = \frac{1-\gamma^2}{\gamma} \left[\frac{2}{5} \right]$$

$$\Rightarrow 3\gamma = 2 - 2\gamma^2 \Rightarrow 2\gamma^2 + 3\gamma - 2 = 0$$

$$\Rightarrow \gamma = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$$

$$\gamma = \frac{1}{2} \text{ or } -2$$

$$\Rightarrow \boxed{\gamma = \frac{1}{2}}$$

3)

The regression equations are $3x + 2y = 26$ and

$6x + y = 31$. Find the correlation coefficient between

x and y .

Given $3x + 2y = 26 \rightarrow \textcircled{1}$

$6x + y = 31 \rightarrow \textcircled{2}$

From $\textcircled{1}$, $2y = -3x + 26$

$y = -\frac{3}{2}x + 13$

$\therefore \boxed{b_{yx} = -\frac{3}{2}}$



From (2) , $6x = -y + 31$
 $x = -\frac{1}{6}y + \frac{31}{6}$

$$\therefore \boxed{b_{xy} = -\frac{1}{6}}$$

The correlation coefficient $\gamma = \pm \sqrt{b_{xy} b_{yx}}$
 $= \pm \sqrt{\left(-\frac{1}{6}\right)\left(-\frac{3}{2}\right)} = \pm \sqrt{\frac{1}{4}}$

$$\boxed{\gamma = -\frac{1}{2}}$$



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