

SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

TOPIC:4.6- MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES

Maxima and Minima

State the condition for f(x,y) be a maximum or minimum.

Maximum

(i) $f_x = 0$ (ii) $f_y = 0$ (iii) $f_{xx} f_{yy} - (f_{xy})^2 > 0$ (iv) $f_{xx} < 0$ (or) $f_{yy} < 0$ (iv) $f_{xx} > 0$ (or) $f_{yy} > 0$



SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

D A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires least materials for its construction. Let x, y, Z be the dimensions of the retangular box. Surface area f(x,y,z) = xy + 2xz + 2gzVolume g(x, y, z) = xyz = 32g(x,y,z) = xyz - 32Hence $F(x,y,z) = \int (x,y,z) + \lambda g(x,y,z)$ $= xy + 2xz + 2yz + \lambda(xyz - 32)$



SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Diff. (1) partially w.r.t.
$$x$$
, y , z .

$$\frac{\partial F}{\partial z} = y + 2z + yz\lambda = 0$$

$$yz\lambda = -y - 2z$$

$$\lambda = -\left[\frac{y + 2z}{yz}\right]$$

$$\lambda = -\left[\frac{1}{z} + \frac{2}{y}\right] \rightarrow (2)$$

$$\frac{\partial F}{\partial y} = x + 2z + xz\lambda = 0$$

$$xz\lambda = -x - 2z$$

$$\lambda = -\left[\frac{x + 2z}{xz}\right]$$

$$\lambda = -\left[\frac{1}{z} + \frac{2}{z}\right] \rightarrow (3)$$



SNS COLLEGE OF ENGINEERING Kurumbapalayam (Po), Coimbatore – 641 107



AN AUTONOMOUS INSTITUTION

Accredited by NBA – AICTE and Accredited by NAAC – UGC with 'A' Grade Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

$$\frac{\partial F}{\partial z} = 2\alpha + 2y + xy\lambda = 0$$

$$2y\lambda = -2x - 2y$$

$$\lambda = -\left[\frac{2x + 2y}{xy}\right]$$

$$\lambda = -\left[\frac{2}{y} + \frac{2}{x}\right] \rightarrow \mathcal{A}$$
From (2) & (3)
$$-\left[\frac{1}{z} + \frac{2}{y}\right] = -\left[\frac{1}{z} + \frac{2}{x}\right]$$

$$\frac{2}{y} = \frac{2}{x} \Rightarrow x = y$$
From (3) & (4),
$$-\left[\frac{1}{z} + \frac{2}{x}\right] = -\left[\frac{2}{y} + \frac{2}{x}\right]$$

$$\frac{1}{z} = \frac{2}{y} \Rightarrow y = 2z$$
Sub. in $g(x, y, z)$

$$xyz = 3z \Rightarrow z^3 = 8 \Rightarrow z = 2$$

$$\Rightarrow 4z^3 = 3z \Rightarrow z^3 = 8 \Rightarrow z = 2$$