TOPIC: 4.4- TAYLOR SERIES FOR FUNCTIONS OF TWO VARIABLES.

Taylor's series for functions of two variables

$$f(x,y) = f(a,b) + \frac{1}{1!}\left[h f_x(a,b) + k f_y(a,b)\right] + \frac{1}{2!}\left[h^2 f_{xx}(a,b) + 2hk f_{xy}(a,b) + k^2 f_{yy}(a,b)\right] + \frac{1}{3!}\left[h^3 f_{xxx}(a,b) + 3h^2 k f_{xxy}(a,b) + 3hk^2 f_{xyy}(a,b) + k^3 f_{yyy}(a,b)\right] + \cdots$$

where $h = x - a$ and $k = y - b$

(2) Expand $e^x \log(1+y)$ in powers of x and y upto terms of third degree.

Function	Value at (0,0)
$f(x,y) = e^x \log(1+y)$	$f = 0$
$f_x = e^x \log(1+y)$ $f_y = e^x \frac{1}{1+y}$	$f_x = 0$ $f_y = 1$
$f_{xx} = e^x \log(1+y)$ $f_{xy} = e^x \cdot \frac{1}{1+y}$ $f_{yy} = -\frac{e^x}{(1+y)^2}$	$f_{xx} = 0$ $f_{xy} = 1$ $f_{yy} = -1$

$f_{xxx} = e^x \log(1+y)$	$f_{xxx} = 0$
$f_{xxy} = \frac{e^x}{1+y}$	$f_{xxy} = 1$
$f_{xyy} = -\frac{e^x}{(1+y)^2}$	$f_{xyy} = -1$
$f_{yyy} = +\frac{2e^x}{(1+y)^3}$	$f_{yyy} = 2$

Here $a = 0$, $b = 0$, $h = x$ and $k = y$.

$$f(x,y) = 0 + \frac{1}{1!}\left[x(0) + y(1)\right] + \frac{1}{2!}\left[x^2(0) + 2xy(1) + y^2(-1)\right]$$

$$+ \frac{1}{3!}\left[x^3(0) + 3x^2y(1) + 3xy^2(-1) + y^3(2)\right]$$

$$= y + \frac{1}{2!}\left[2xy - y^2\right] + \frac{1}{3!}\left[3x^2y - 3xy^2 + 2y^3\right]$$