TOPIC: 4.7 MAXIMA AND MINIMA OF FUNCTIONS OF TWO VARIABLES

2) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 squar meter.

Let x, y, z be the rectangular box dimension

volume $f(x, y, z) = xyz$

surface area $g(x, y, z) = xy + 2yz + 2xz = 432$

$$g(x, y, z) = xy + 2yz + 2xz - 432$$

Hence $F(x, y, z) = f(x, y, z) + \lambda\, g(x, y, z)$

$$= xyz + \lambda(xy + 2yz + 2xz - 432) \longrightarrow (1)$$

Diff. (1) partially w.r.t 'x', 'y' and 'z'

$$\frac{\partial F}{\partial x} = yz + \lambda(y + 2z) = 0$$

$$\lambda(y+2z) = -yz$$

$$\lambda = \frac{-yz}{y+2z} \longrightarrow (2)$$

$$\frac{\partial F}{\partial y} = xz + \lambda(x + 2z) = 0$$

$$\lambda(x+2z) = -xz$$

$$\lambda = \frac{-xz}{x+2z} \longrightarrow (3)$$

$$\frac{\partial F}{\partial z} = xy + \lambda(2y + 2x) = 0$$

$$\lambda(2y+2x) = -xy$$

$$\lambda = -\frac{xy}{2y+2x} \longrightarrow (4)$$

From (2) & (3),

$$\frac{-yz}{y+2z} = \frac{-xz}{x+2z} \Rightarrow \frac{y}{y+2z} = \frac{x}{x+2z}$$

$$\Rightarrow xy + 2yz = xy + 2xz$$

$$\Rightarrow 2yz = 2xz \Rightarrow \boxed{x = y}$$

From ③ & ④,

$$-\frac{xz}{x+2z} = -\frac{xy}{2y+2x} \Rightarrow \frac{z}{x+2z} = \frac{y}{2y+}$$

$$\Rightarrow 2yz + 2xz = xy + 2yz$$

$$\Rightarrow 2xz = xy \Rightarrow \boxed{y = 2z}$$

$$\therefore \boxed{x = y = 2z}$$

sub. in $g(x,y,z)$

$$xy + 2yz + 2xz = 432$$

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$$xy + 2yz + 2xz = 432$$

$$(2z)(2z) + 2(2z)z + 2(2z)z = 432$$

$$\Rightarrow 4z^2 + 4z^2 + 4z^2 = 432$$

$$\Rightarrow 12z^2 = 432 \Rightarrow z^2 = 36$$

$$\Rightarrow \boxed{z = 6}$$

$$\therefore \boxed{x = 12} \quad \boxed{y = 12}$$