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**Topic:4.9–LAGRANGE'S METHOD**

Using Lagrange's method of multipliers,  
show that the stationary value of  $a^3 x^2 + b^3 y^2 + c^3 z^2$   
where  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$  occur at  $x = \frac{a+b+c}{a}$  ;  
 $y = \frac{a+b+c}{b}$  ;  $z = \frac{a+b+c}{c}$  ;

Solution: Let the Auxiliary function F be  
 $F(x, y, z, \lambda) = (a^3 x^2 + b^3 y^2 + c^3 z^2) + \lambda \left( \frac{1}{x} + \frac{1}{y} + \frac{1}{z} - 1 \right)$   
where  $\lambda$  is Lagrange multiplier.

$$F_x = \frac{\partial F}{\partial x} = 2a^3 x - \frac{\lambda}{x^2}$$
$$F_y = \frac{\partial F}{\partial y} = 2b^3 y - \frac{\lambda}{y^2}$$
$$F_z = \frac{\partial F}{\partial z} = 2c^3 z - \frac{\lambda}{z^2}$$



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To find the stationary value.

$$F_x = 0 \quad F_y = 0 \quad F_z = 0$$
$$2a^3x - \frac{\lambda}{x^2} = 0 \quad 2b^3y - \frac{\lambda}{y^2} = 0 \quad 2c^3z - \frac{\lambda}{z^2} = 0$$
$$2a^3x = \frac{\lambda}{x^2} \quad 2b^3y = \frac{\lambda}{y^2} \quad 2c^3z = \frac{\lambda}{z^2}$$
$$a^3x^3 = \frac{\lambda}{2} \quad by^3 = \frac{\lambda}{2} \quad c^3z^3 = \frac{\lambda}{2}$$
$$ax = \left(\frac{\lambda}{2}\right)^{\frac{1}{3}} \quad by = \left(\frac{\lambda}{2}\right)^{\frac{1}{3}} \quad cz = \left(\frac{\lambda}{2}\right)^{\frac{1}{3}}$$

$\hookrightarrow$  ①                       $\hookrightarrow$  ②                       $\hookrightarrow$  ③

from ①, ②, ③, we get.  
 $ax = by = cz$ .

Given  $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ .

$$\frac{a}{ax} + \frac{b}{by} + \frac{c}{cz} = 1 \quad [\text{since } ax = by = cz]$$

$$\frac{a+b+c}{ax} = 1 \Rightarrow x = \frac{a+b+c}{a}$$

$$y = \frac{a+b+c}{b} \quad ; \quad z = \frac{a+b+c}{c}$$

$\therefore$  f is stationary at this point.



3) Find the point on the plane  $ax + by + cz = p$  at which  $f = x^2 + y^2 + z^2$  has a stationary value and find the stationary value of  $f$ , using Lagrange's method of multipliers,

Sol: Let the auxiliary function  $F$  be

$$F(x, y, z, \lambda) = x^2 + y^2 + z^2 + \lambda(ax + by + cz - p)$$

where  $\lambda$  is Lagrange multiplier,

$$F_x = \frac{\partial F}{\partial x}$$

$$F_y = \frac{\partial F}{\partial y}$$

$$F_z = \frac{\partial F}{\partial z}$$

$$= 2x + \lambda a$$

$$= 2y + \lambda b$$

$$= 2z + \lambda c$$

To find the stationary values,

$$F_x = 0$$

$$2x + \lambda a = 0$$

$$2x = -\lambda a$$

$$F_y = 0$$

$$2y + \lambda b = 0$$

$$2y = -\lambda b$$

$$F_z = 0$$

$$2z + \lambda c = 0$$

$$2z = -\lambda c$$



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$$\frac{x}{a} = -\frac{\lambda}{2} \rightarrow \textcircled{1}$$

$$\frac{y}{b} = -\frac{\lambda}{2} \rightarrow \textcircled{2}$$

$$\frac{z}{c} = -\frac{\lambda}{2} \rightarrow \textcircled{3}$$

from (1), (2), (3) we get.

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

$$\frac{ax}{a^2} = \frac{by}{b^2} = \frac{cz}{c^2}$$

$$\Rightarrow \frac{ax}{a^2} = \frac{by}{b^2} = \frac{cz}{c^2} = \frac{ax + by + cz}{a^2 + b^2 + c^2} = \frac{P}{a^2 + b^2 + c^2}$$

$$x = \frac{aP}{a^2 + b^2 + c^2} ; y = \frac{bP}{a^2 + b^2 + c^2} ; z = \frac{cP}{a^2 + b^2 + c^2}$$



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$$\begin{aligned} \text{stationary value of } f &= x^2 + y^2 + z^2, \\ &= \left( \frac{ap}{a^2 + b^2 + c^2} \right)^2 + \left( \frac{bp}{a^2 + b^2 + c^2} \right)^2 + \left( \frac{cp}{a^2 + b^2 + c^2} \right)^2 \\ &= \frac{a^2 p^2 + b^2 p^2 + c^2 p^2}{(a^2 + b^2 + c^2)^2} \\ &= \frac{(a^2 + b^2 + c^2) p^2}{(a^2 + b^2 + c^2)^2} \\ &= \frac{p^2}{a^2 + b^2 + c^2} \end{aligned}$$

We can prove this to be minimum,



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