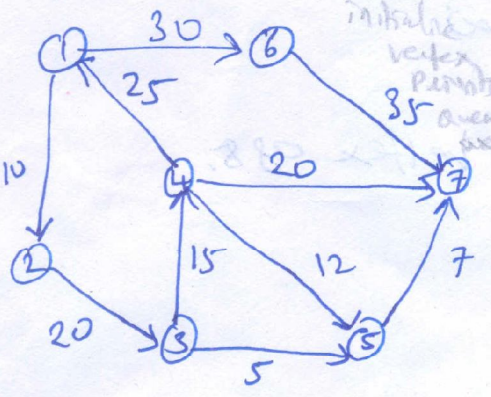
Single source shortest path method:- Dijkstra's algorithm

Many times, graph is used to represent the distance b/w two cities every body is often interested in moving from one city to other as quickly as possible. The single source shortest path is based on this interest.

In single source shortest path problem the shortest distance from a single vertex called source is obtained.

Let G(V, E) be a graph, then in single source shortest path the shortest paths from vertex v_0 to all remaining vertex is determined. The vertex v_0 is then called as source & last vertex is called destination.

eg:-



we will start from source vertex 1, hence set S[1] = 1

Now shortest distance from vertex 1 is 10. ie. 1 → 2 = 10

hence {1, 2} & min = 10.

from vertex 2 the next

$\{1,5\} = \infty$

$\{1,6\} = \infty 30$

$\{1,7\} = \infty$

now

$\{1,2,3\} = 30$

$\{1,2,4\} = \infty$

$\{1,2,7\} = \infty$

$\{1,2,5\} = \infty$

$\{1,2,6\} = \infty$

hence select 3.

$S[3] = 1$

:

now $\{1,2,3,4\} = 45$

$\{1,2,3,5\} = 35$

$\{1,2,3,6\} = \infty$

$\{1,2,3,7\} = \infty$

hence ~~next~~ vertex as 5.

$\therefore S[5] = 1$

now

$\{1,2,3,5,6\} = \infty$

$\{1,2,3,5,7\} = 42$

hence vertex 7 will be selected. In single

1, 2	10
1, 2, 3	30
1, 2, 3, 4	45
~~1, 2, 3, 4~~	~~35~~
1, 2, 3, 5	35
1, 2, 3, 4, 7	45
1, 2, 3, 5, 7	32
1, 6	30
1, 6, 7	65

Algorithm:-

```
Single_Shortest_path (P, cost, Dist, n)
{
 for i ← 1 to n do
 {
  S[i] ← 0;
  Dist ← cost[P, i];
 }
 S[P] ← 1  -> Set pth vertex to true in array S & i.e. put P in S
 Dist[P] ← 0.0;
 for val ← 2 to n-2 do
 { // obtain n-1 paths from P
```

update the distance values of other nodes

```
for (all node r adjacent to q with S[r] = 0) do
  if (Dist[r] > (Dist[q] + Cost[p,q])) then
    Dist[r] ← Dist[q] + Dist[p,q];
}
```

Analysis:-

Time complexity of algorithm

- 1st for-loop clearly takes O(n) time
- choosing q takes O(n) time, because it involves finding a minimum in array.
- innermost for-loop for updating Dist iterates at most n times, thus takes O(n) time.

∴ Single source shortest path takes $O(n^2)$ time.
