i = 0	0					
i = 1	0	10	12	12	12	12
i = 2	0	10	22	22	22	22
i = 3	0					
i = 4	0					

$$V(1,1) = \max\{v(i-1,j),\ v_i + v[i-1, j-w_i]\}$$

$$= \max\{V(1-1,1),\ 12 + v[1-1, 1-2]\}$$

$$= \max\{V(0,1),\ 12 + V[0,-1]\}$$

$$= \max\{0,\ 12 + V[0]\}$$

$$= 12$$

$$V(1,2) = \max\{V[1-1,2),\ 12 + V[1-1, 2-2]\}$$

$$= \max\{V[0,2),\ 12 + V[0,0)\}$$

$$= \max\{0,\ 12 + 0\}$$

$$= 12$$

$$V[1,3) = \max\{V[1-1,3),\ 12 + V[1-1, 3-2]\}$$

$$= \max\{V(0,3),\ 12 + V(0,1)\}$$

$$= \max\{0,\ 12 + 0\}$$

$$= 12$$

$$V[1,4) = \max\{V[1-1,4),\ 12 + V[1-1, 4-2]\}$$

$$= \max\{V[0,4),\ 12 + V[0,2)\}$$

$$= \max\{0,\ 12 + 0\}$$

$$= 12$$

Knapsack problem & memory function

given n items of known weights $w_1 \ldots w_n$ & values $v_1 \ldots v_n$ & knapsack capacity W, find most valuable subset of items that fill into knapsack.

dp algorithm, derive recurrence relation item i, $1 \le i$ weights $w_1 \ldots w_i$ value $v_1 \ldots v_i$ & knapsack capac $1 \le j \le W$

let $v(i,j)$ value of optimal solution

1) among subsets that do not include ith item, value of an optimal subset is by definition $V[i-1, j]$.

2) Among subsets that do include ith item, an opt subset is made up of this item & optimal subset i-1 items fit into knapsack capacity $j - w_i$. value of optimal subset is $v_i + V[i-1, j-w_i]$

$$V[i,j] = \begin{cases} \max\{V[i-1,j],\ v_i + V[i-1, j-w_i]\} & \text{if } j - w_i \\ V[i-1, j] & \text{if } j - w_i < \end{cases}$$

	0	$j-w_i$	j	W
0	0	0	0	0
i-1	0	$V[i-1, j-w_i]$	$V[i-1, j]$	
i	0		$V[i,j]$	
n	0			goal

Table for solving knapsack

	v_i	w_i
1	15	1
2	10	5
3	9	3
4	5	4

1 2 $\{21, 5V_{22}\} = 3$

			item	weight	value
1 3	5	22			
1 4	4	27	1	2	\$12
2 3 / 2 1	4	30	2	1	\$10
	3	25	3	3	\$20
1 2 7	6		4	2	\$15

capacity $W = 5$

$i=0$	0	0	0	0	0	0
$i=1$	0	10	12	12	12	12
$i=2$	0	10	22	22	22	22
$i=3$	0					
$i=4$	0					

$$V(1,1) = \max\{v(i-1,j),\ v_i + v[i-1,\ j-w_i]\}$$

$$= \max\{v(1-1,1),\ 12 + v[1-1,\ 1-2]\}$$

$$= \max\{v(0,1),\ 12 + v[0,\ -1]\}$$

$$= \max\{0,\ 12 + v[0]\}$$

$$= 12$$

$$v(1,2) = \max\{v[1-1,\ 2),\ 12 + v[1-1,\ 2-2]\}$$

$$= \max\{v[0,2),\ 12 + v[0,\ 0)\}$$

$$= \max\{0,\ 12 + 0\}$$

$$= 12$$

$$v[1,3) = \max\{v(1-1,\ 3),\ 12 + v[1-1,\ 3-2]\}$$

$$= \max\{v(0,3),\ 12 + v(0,\ 1)\}$$

$$= \max\{0,\ 12 + 0\}$$

$$= 12$$

$$v(1,4) = \max\{v[1-1,4),\ 12 + v[1-1,\ 4-2]\}$$

$$= \max\{v[0,4),\ 12 + v[0,\ 2)\}$$