Dynamic programming

computing Binomial coefficient

Combinatorics i.e binomial co-efficient denoted.

$C(n,k)$ or $\binom{n}{k}$, no of combinations (subsets) of k elem from n-element set ($0 \le k \le n$).

Binomial formula:-

$$(a+b)^n = C(n,0)a^n + \dots + C(n,k)a^{n-k}b^k + \dots + C(n,n)b^n.$$

we concrete into two:

$$C(n,k) = C(n-1,k-1) + C(n-1,k) \quad \text{for } n>k>0$$

&

$$C(n,0) = C(n,n) = 1$$

recurrence computing $C(n,k)$ - smaller & overlapping problems of computing $C(n-1,k-1)$ & $C(n-1,k)$

To do this problem - Binomial coefficients in table

n+1 rows & k+1 columns no from 0 to n & 0 to k

$$\sum_{i=1}^{k}\sum_{j=1}^{p-1} + \sum_{i=k+1}^{n}\sum_{j=p}^{m}$$

i\j	0	1	2	j-1	k-1	k
0	1					
1	1	1				
2	1	2	1			
⋮						
k	1				1	
n-1	1				$C(n-1,k-1)$	$C(n-1,k)$
n	1					$C(n,k)$

i\j	1	2	…	p-1	p	…	m
1							
2							
3							
⋮							
k							
k+1							
⋮							
n-1							
n							

-2 -1 0 1 2

0 k n

Alg Binomial(n, k)

// compute ...

$$\cancel{A(n,k) = \sum_{i=1} \sum_{j=1} 1 + \sum_{i=k+1} \sum_{j=1} 1 = \sum_{i=1} (i-1) + \sum_{i=k+1} k}$$

$$= \frac{(k-1)k}{2} + k(n-k) \in \Theta(nk)$$

$$\frac{k}{2} + nk - 2k \quad nk$$

warshall's algorithm

~~transitive closure of directed graph~~

A ~~djacency matrix~~

$$A(n,k) = \sum_{i=1}^{k} \sum_{j=1}^{i-1} 1 + \sum_{i=k+1}^{n} \sum_{j=1}^{k} 1$$

$$= \sum_{i=1}^{k} (i-1) + \sum_{i=k+1}^{n} k$$

n, k

$4, 2$

$C(n,k) = C(n-1, k-1) + C(n-1, k)$

$C(4,2) = C(3,1) + C(3,2) \rightarrow 1$

2 unknown values : $C(3,1)$ & $C(3,2)$ for calc $C(4,2)$

$\therefore n=3 \quad k=1$

$C(3,1) = C(2,0) + C(2,1)$

As $C(n,0) = 1$ we can write

$C(2,0) = 1$

$C(3,1) = 1 + C(2,1) \rightarrow 2$

Compute $C(2,1)$

$n=2 \quad k=1$

$C(2,1) = C(1,0) + C(1,1)$

$\therefore C(n,0) = 1 \Rightarrow C(1,0) = 1 \Rightarrow C(n,n) = 1 \Downarrow C(1,1) = 1$

$C(2,1) = 1 + C(1,1)$

$= 1 + 1$

$= 2$

$C(2,1) = 2 \rightarrow 3$

solve eqn 3 to 2

$C(3,2) = C(2,1) + C(2,2)$

$\boxed{C(2,1) = 2} \rightarrow 3$

$C(2,2) = 1 \Rightarrow C(n,n) = 1$

$\therefore C(3,2) = 2 + 1$

$= 3 \rightarrow 5$

$\therefore C(4,2) = C(3,1) + C(3,2)$

$= 3 + 3$

$= 6 \rightarrow$ final answer.

$\sum_{i=1}^{n} x$ $(1+2+3+\dots n) = \frac{n(n+1)}{2}$

$\sum_{i=1}^{k-1} (i-1)$ $1+2+\dots+k-1 = \frac{(k-1)(k+1)}{2}$

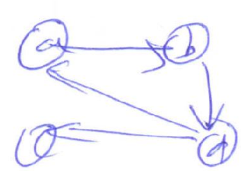
Warshall's Algorithm.

Transitive closure of directed graph with n vertices can be defined as n-by n boolean matrix $T = \{t_{ij}\}$.

element in i^{th} row & j^{th} column

$(1 \le i \le n)$ $(1 \le j \le n)$

directed graph.



adjacency matrix

$$A = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 0 & 0 & 0 & 1 \\ c & 0 & 0 & 0 & 0 \\ d & 1 & 0 & 1 & 0 \end{array}$$

Transitive closure

$$T = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 1 & 1 & 1 & 1 \\ c & 0 & 0 & 0 & 0 \\ d & 1 & 1 & 1 & 1 \end{array}$$

$R^{(0)} \dots R^{(k-1)}, R^{(k)} \dots R^{(n)}$

$r_{ij}^{(k)} = r_{ij}^{(k-1)}$

$\rightarrow$ if an element r_{ij} is 1 in $R^{(k-1)}$, it remains 1 in R^{k}