



Topic:5.6 – EULERS(CAUCHYS) LINEAR EQUATIONS

An equation of the form

$$x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \dots + a_n y = f(x) \rightarrow \textcircled{1}$$

where a_1, a_2, \dots, a_n are constants and $f(x)$ is a function of x .

\textcircled{1} can be reduced to linear differential equation with constant coefficient by putting the sub.

$$x = e^z \quad (\text{or}) \quad z = \log x$$

$$x \frac{dy}{dx} = D' y \quad \text{where} \quad D' = \frac{d}{dz}$$

$$x^2 \frac{d^2 y}{dx^2} = D'(D'-1) y$$

$$x^3 \frac{d^3 y}{dx^3} = D'(D'-1)(D'-2) y$$



1) solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$

Given $(x^2 D^2 + xD) y = 0$

Put $x = e^z$ $\log x = z$

$x D = D'$ $x^2 D^2 = D'(D'-1)$

$[D'(D'-1) + D'] y = 0$

$[D'^2 - D' + D'] y = 0 \Rightarrow D'^2 y = 0$

AE $m^2 = 0$; $m = 0, 0$

CF = $[A + Bz] e^{0z}$

= $A + Bz$

CF = $A + B \log x$

$y = A + B \log x$ [\because RHS = 0 PI = 0]



2) solve: $(x^2 D^2 - 2x D - 4)y = x^2 + 2 \log x$

Put $x = e^z$ $\log x = z$

$x D = D'$ $x^2 D^2 = D'(D'-1)$

$[D'(D'-1) - 2D' - 4]y = e^{2z} + 2z$

$[D'^2 - D' - 2D' - 4]y = e^{2z} + 2z$

$[D'^2 - 3D' - 4]y = e^{2z} + 2z$

AE $m^2 - 3m - 4 = 0 \Rightarrow (m-4)(m+1) = 0$

$m = -1, 4$

CF = $Ae^{-z} + Be^{4z}$

$= Ax^{-1} + Bx^4 = \frac{A}{x} + Bx^4$

$PI_1 = \frac{1}{D'^2 - 3D' - 4} e^{2z} = \frac{1}{4 - 6 - 4} e^{2z}$

$= \frac{1}{-6} e^{2z} = -\frac{1}{6} x^2$

$PI_2 = \frac{1}{D'^2 - 3D' - 4} 2z = 2 \frac{1}{-4 \left(1 + \frac{3D' - D'^2}{4}\right)} z$



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$$\begin{aligned} &= \frac{1}{-6} e^{2z} = -\frac{1}{6} x^2 \\ PI_2 &= \frac{1}{D'^2 - 3D' - 4} 2z = 2 \frac{1}{-4 \left(1 + \frac{3D' - D'^2}{4}\right)} z \\ &= -\frac{1}{2} \left[1 + \frac{3D' - D'^2}{4}\right]^{-1} z \\ &= -\frac{1}{2} \left[1 - \left(\frac{3D' - D'^2}{4}\right) + \dots\right] z \\ &= -\frac{1}{2} \left[1 - \frac{3D'}{4} + \frac{D'^2}{4}\right] z \\ &= -\frac{1}{2} \left[z - \frac{3}{4}\right] = -\frac{z}{2} + \frac{3}{8} \end{aligned}$$

$$PI_2 = -\frac{1}{2} \log x + \frac{3}{8}$$

$$\therefore y = \frac{A}{x} + Bx^4 - \frac{1}{6}x^2 - \frac{1}{2} \log x + \frac{3}{8}$$



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