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AN AUTONOMOUS INSTITUTION

Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai.

UNIT – II WAVES AND OPTICS

TOPIC- III EINSTEIN’S COEFFICIENT

5.1 INTRODUCTION

The word "**laser**" is an acronym for Light Amplification by Stimulated Emission of Radiation. Lasers have many important applications. They are used in common consumer devices such as DVD players, laser printers, and barcode scanners.

They are used in medicine for laser surgery and various skin treatments, and in industries for cutting and welding materials.

5.2 CHARACTERISTICS OF LASER

(i) Directionality

Ordinary light spreads in all directions and its angular spread is 1 metre/metre. But it is found that laser is highly directional and its angular spread its 1mm/metre.

(ii) Intensity

An ordinary light spreads in all directions; the intensity reaching the target is very less. But in the case of laser, due to high directionality the intensity of laser beam reaching the target is of high intense beam.

For example, 1 milli watt power of He-Ne laser appears to be brighter than the sunlight.

(iii)Monochromaticity

Laser beam is highly monochromatic i.e. the wavelength is single, whereas in ordinary light like mercury vapour lamp, many wavelengths of light are emitted.

(iv)Coherence

The light from a laser is said to be highly coherent, which means that the waves of laser light are in same amplitude and phase. There are two types of coherence,

- temporal coherence
- spatial coherence.

5.3 PRINCIPLE OF SPONTANEOUS AND STIMULATED EMISSION EINSTEIN’S QUANTUM THEORY OF RADIATION

When light is absorbed by the atoms or molecules, then it goes from the lower energy level (E_1) to the higher energy level (E_2) and during the transition from higher energy level (E_2) to lower energy level (E_1), the light is emitted from the atoms or molecules. Let us consider an atom exposed to (light) photons of energy E_2-E_1= h^{γ}, three distinct processes takes place.

i. Absorption

ii. Spontaneous emission

iii. Stimulated emission

i. Absorption

An atom in the lower energy level or ground state energy level E_1absorbs the incident photon radiation of energy (h^γ) and goes to the higher energy level or excited energy state E_2 as shown in fig 5.1. This process is called as absorption.

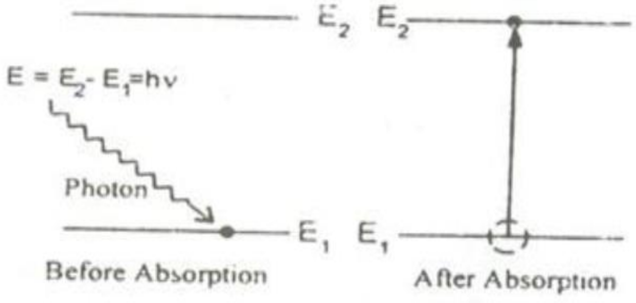

Fig. 5.1

If there are many number of atoms in the ground state then each atom will absorb the energy from the incident photon and goes to the excited state then,

The rate of absorption (R_{12}) is proportional to the following factors.

i.e. $R_{12}\ \alpha$ Energy density of incident radiation (ρ_v)

α No of atoms in the ground state (N_1)

i.e. $R_{12}\ \alpha\ \rho_v N_1$

or $R_{12} = B_{12}\,\rho_v N_1$ ----(1)

Where, B_{12} is a constant which gives the probability of absorption transition per unit time.

ii. Spontaneous emission

The atom in the excited state returns to the ground state by emitting a photon of energy E= (E_2– E_1) = h^γ, spontaneously without any external triggering as shown in figure 5.2.

This process is known as spontaneous emission. Such an emission is random and is independent of incident radiation.

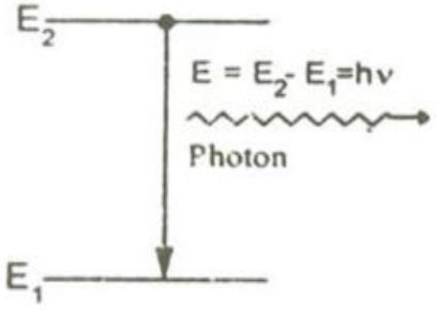

Fig. 5.2

If N_1 and N_2 are the numbers of atoms in the ground state (E_1) and excited state (E_2) respectively, then

The rate of spontaneous emission is $R_{21}(Sp) \alpha N_2$

$$(\text{or})\ R_{21}(Sp) = A_{21}N_2 \qquad \text{-----(2)}$$

Where, A_{21} is a constant which gives the probability of spontaneous emission transition per unit time.

iii. Stimulated emission

The atom in the excited state can also return to the ground state by external triggering (or) inducement of photon there by emitting a photon of energy equal to the energy of the incident photon, known as stimulated emission. Thus results in two photons of same energy, phase difference and of same directionality as shown in figure5.3.

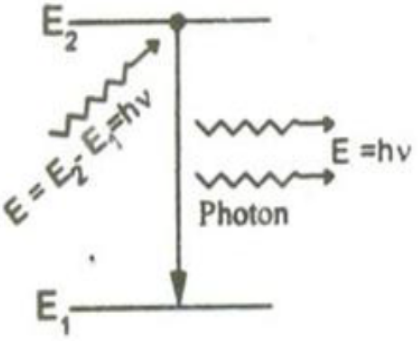

Fig.5.3

The rate of stimulated emission is $R_{21}(St) \alpha \rho_v N_2$

$$(\text{or}) \qquad R_{21}(St) = B_{21}\rho_v N_2 \qquad \text{----(3)}$$

Where, B_{21} is a constant which gives the probability of stimulated emission transition per unit time.

Einstein's theory

Einstein's theory of absorption and emission of light by an atom is based on Planck's theory of radiation. Also under thermal equilibrium, the population of energy levels obeys the Boltzmann's distribution law.

i.e. under thermal equilibrium

The rate of absorption = The rate of emission

$$B_{12}\rho_v N_1 = A_{21}N_2 + B_{21}\rho_v N_2$$

$$\rho_v [B_{12}N_1 - B_{21}N_2] = A_{21}N_2$$

$$\therefore \rho_v = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$\therefore \rho_v = \frac{A_{21}}{B_{12}({N_1}/{N_2}) - B_{21}} \qquad \text{----(4)}$$

We know from Boltzmann distribution law

$$N_1 = N_0 e^{-E_1/K_BT}$$

Similarly

$$N_2 = N_0 e^{-E_2/K_BT}$$

Where

K_B - Boltzmann Constant

N_0 - Number of atoms at absolute zero

T- Absolute temperature

At equilibrium, we can write the ratio of population levels as follows,

$$\frac{N_1}{N_2} = e^{(E_2-E_1)/K_B T}$$

Since $E_2 - E_1 = h\vartheta$, we have

$$\frac{N_1}{N_2} = e^{h\vartheta/K_B T} \qquad \text{----(5)}$$

Sub eqn (5) in (4), we get

$$\rho_v = \frac{A_{21}}{B_{12}(e^{h\vartheta/K_B T}) - B_{21}}$$

$$\rho_v = \frac{A_{21}}{B_{21}} \frac{1}{(B_{12}/B_{21}) e^{h\vartheta/K_B T} - 1} \qquad \text{----(6)}$$

This equation has a very good agreement with plank's energy distribution radiation law

$$\rho_v = \frac{8\pi h\vartheta^3}{C^3} \frac{1}{e^{h\vartheta/K_B T} - 1} \qquad \text{---(7)}$$

Therefore comparing (6) and (7), we have

$$B_{12} = B_{21} = B$$

and

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\vartheta^3}{C^3} \qquad \text{----(8)}$$

Taking $A_{21} = A$

The constants A and B are called as Einstein Coefficients, which accounts for spontaneous and stimulated emission probabilities. Ratio of magnitudes of stimulated and spontaneous emission rates are as follows,

From eqn (2) and (3) we have

$$\frac{R_{21}(st)}{R_{21}(sp)} = \frac{B_{21}\rho_v N_2}{A_{21} N_2}$$

$$\frac{R_{21}(st)}{R_{21}(sp)} = \frac{B_{21}\rho_v}{A_{21}} \qquad \text{---(9)}$$

Rearranging eqn (6) we can write

$$\frac{B_{21}\rho_v}{A_{21}} = \frac{1}{(B_{12}/B_{21}) e^{h\vartheta/K_B T} - 1}$$

Since$B_{12} = B_{21}$, we have

$$\frac{1}{e^{h\theta/K_BT-1}} = \frac{B_{21}\rho_v}{A_{21}} \qquad \text{--- (10)}$$

Comparing (9) and (10) we get

$$\frac{R_{21}(st)}{R_{21}(sp)} = \frac{1}{e^{h\theta/K_BT-1}} = \frac{B_{21}\rho_v}{A_{21}}$$

In simpler way the ratio can be written as

$$R = \frac{B_{21}\rho_v}{A_{21}}$$

Generally spontaneous emission is more predominant in the optical region (ordinary light). To increase the number of coherent photons stimulated emission should dominate over spontaneous emission.

5.4 DIFFERENCE BETWEEN SPONTANEOUS AND STIMULATED EMISSION OF RADIATION

S. No	**Stimulated emission**	**spontaneous emission**
1.	An atom in the excited state is induced to return to ground state, thereby resulting in two photons of same frequency and energy is called stimulated emission.	The atom in the excited state returns to ground state thereby emitting a photon, without any external inducement is called spontaneous emission.
2.	The emitted photons move in same direction and is highly directional	The emitted photons move in all directions and are random.
3.	The radiation is high intense, monochromatic and coherent.	The radiation is less intense and is incoherent.
4.	The photons are in phase (i.e.) there is a constant phase difference.	The photons are not in phase (ie.) there is no phase relationship between them.
5.	The rate of transition is given by $R_{21}(St) = B_{21}\rho_v N_2$	The rate of transition is given by $R_{21}(Sp) = A_{21}N_2$