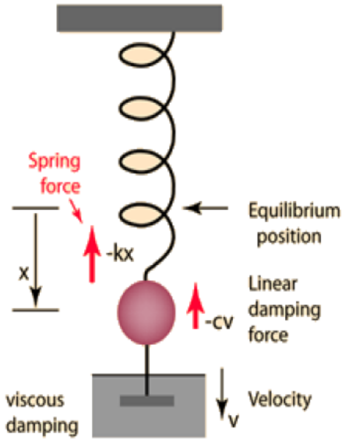
UNIT – II WAVES AND OPTICS

TOPIC – I OSCILLATORY MOTION

Let us consider a mass 'm' connected to a spring and an external force is applied. Here 3 type of forces act on it.

1. A restoring force F1 = - ky
2. Damping Force F2 = -r $\frac{dy}{dt}$ &
3. External force F3 = $F_0 \sin \omega t$



Total Force F = F1 + F2 + F3

$$F = -ky - r\frac{dy}{dt} + F0 \sin \omega t \quad \rightarrow (1)$$

From Newton's II law

F = ma

$$F = m\frac{d^2y}{dt^2} \quad \rightarrow (2)$$

Equating (2) and (3),

$$m\frac{d^2y}{dt^2} = -ky - r\frac{dy}{dt} + F0 \sin \omega t$$

$$\frac{d^2y}{dt^2} = -\frac{ky}{m} - \frac{r}{m}\frac{dy}{dt} + \frac{Fo}{m}\sin \omega t = 0$$

$$\frac{d^2y}{dt^2} + \frac{k}{m}y + \frac{r}{m}\frac{dy}{dt} + = \frac{Fo}{m}\sin \omega t \rightarrow (3)$$

Let $\frac{r}{m}$ = 2b, $\frac{k}{m} = \omega_0^2$ & $\frac{F0}{m}$ = f

Therefore Equation 3 is written as,

$$\frac{d^2y}{dt^2} + \omega_0^2 y + 2b\frac{dy}{dt} + = f \sin \omega t \rightarrow (4)$$

Equation is II order differential equation and the solution is,

$y = A \sin(\omega t - \theta) \rightarrow (5)$

where, A = Amplitude

θ = Angle

Differential Equation (5) with respect to time twice,

$$\frac{dy}{dt} = A\cos(\omega t - \theta)\omega \rightarrow (6)$$

$$\frac{d^2y}{dt^2} = -A \sin(\omega t - \theta)\omega^2 \rightarrow (7)$$

Substitute (5) (6) and (7) in (4)

$$-A \sin(\omega t - \theta)\omega^2 + 2b\,A\cos(\omega t - \theta)\omega + \omega_0^2 A \sin(\omega t - \theta) = f \sin[(\omega t - \theta) + \theta]$$

(or)

$$A(\omega_0^2 - \omega^2)\sin(\omega t - \theta) + 2b\omega A\cos(\omega t - \theta) = f \sin(\omega t - \theta)\cos\theta + f\cos(\omega t - \theta)\sin\theta \rightarrow (8)$$

Therefore equation (8) holds good for all values of "t"

Therefore coefficients of $\sin(\omega t - \theta)$ & $\cos(\omega t - \theta)$ must be equal on both sides

Therefore

$$A(\omega_0^2 - \omega^2) = f\cos\theta \rightarrow (9)$$

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Similarly, we can write,

$$2b\omega A = f\sin\theta \rightarrow (10)$$

Squaring and adding equation (9) and (10) we get,

$$A^2(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2A^2 = f^2(\cos^2\theta + \sin^2\theta)$$

Or

$$A^2[(\omega_0^2 - \omega^2)^2 + 4b^2\omega^2] = f^2$$

$$A = \frac{f}{(\omega_0^2 - \omega^2)2 + 4b2\omega^2} \rightarrow (11)$$

Dividing equation (10) by (9) we get,

$$\frac{2b\omega A}{A(\omega_0^2 - \omega^2)} = \frac{f\sin\theta}{f\cos\theta}$$

or

$$\tan\theta = \frac{2b\omega}{(\omega_0^2 - \omega^2)}$$

or

$$\theta = \tan^{-1}\left[\frac{2b\omega}{(\omega_0^2 - \omega^2)}\right] \rightarrow (12)$$

Special cases:

Case 1 : When $\omega_0^2 \gg \omega^2$

Amplitude $A = \frac{f0}{K}$

Phase $\theta = 0$

Therefore displacement and dividing force will be in phase.

Case 2: When $\omega_0^2 = \omega^2$ => Resonant Frequency

Amplitude $A = \frac{f0}{\omega}$

Phase $\theta = \frac{\pi}{2}$

Therefore displacement lags behind the dividing force by 90^0 (or) $\frac{\pi}{2}$

Case 3: When $\omega_0^2 \ll \omega^2$

Amplitude $A = \frac{f0}{m\omega^2}$

Phase $\theta = \pi$

Therefore displacement lags behind the dividing force by 180^0 (or) π