SNS COLLEGE OF ENGINEERING

(Autonomous)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

UNIT- II

IIR Filter Design

Analog Filters-Butterworth filter

ANALOG FILTER DESIGN:

The most general form of analog filter transfer fn H(S),

$$H(S) = \frac{N(S)}{D(S)}$$

$$H(S) = \frac{\sum_{i=0}^{M} b_i s^i}{1 + \sum_{i=1}^{N} a_i s^i}$$

where,

H(S) is the Laplace transform of the impulse response h(t),

i.e., $H(S) = \int_{-\infty}^{\infty} h(t) \cdot e^{-St} \cdot dt$.

It should satisfy the condition,

$N \geq M$ ...

For a stable analog filter the poles of H(S) should lie in left half of S-plane.

TYPES OF ANALOG FILTER:

i) Butterworth filter

ii) Chebyshev filter.

(ANALOG LOW PASS BUTTERWORTH FILTER:

Let the maximum pass band attenuation in positive decibels (dB) is α_p (< 3 dB). At pass band frequency Ω_p and α_s in the minimum stop band attenuation in +ve dB at stop band frequency Ω_s.

$$\therefore |H(j\Omega)| = \frac{1}{\left[1 + \varepsilon^2 \left(\frac{\Omega}{\Omega_p}\right)^{2N}\right]^{1/2}} \quad \text{——— (I)}$$

Epsilon

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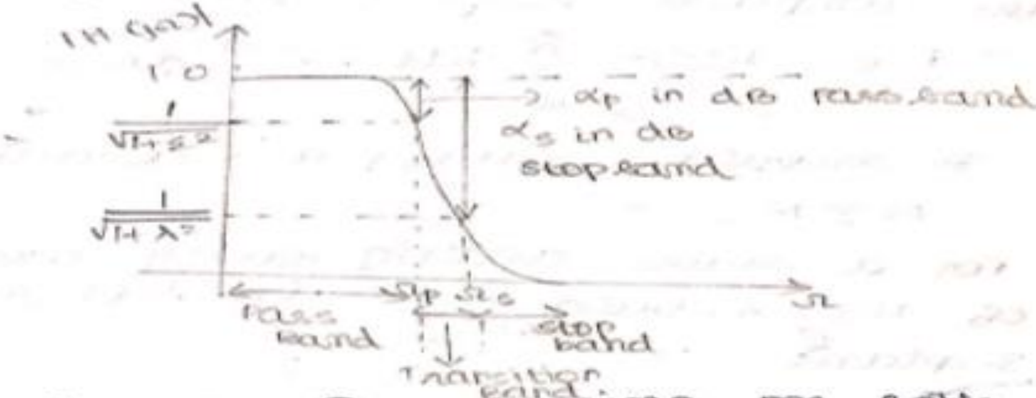
④ derivation of Butterworth Approximation of Magnitude response :



From eqn ①, squaring on both sides,

$$|H(j\Omega)|^2 = \frac{1}{\left[1+\varepsilon^2\left(\Omega/\Omega_p\right)^{2N}\right]}$$

Taking log on both sides & multiplying by 10,

$$10\log|H(j\Omega)|^2 = 10\log(1) - 10\log\left[1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right]$$

$$20\log|H(j\Omega)| = (0) - 10\log\left[1+\varepsilon^2\left[\frac{\Omega}{\Omega_p}\right]^{2N}\right]$$

$$20\log|H(j\Omega)| = -10\log\left[1+\varepsilon^2\left(\frac{\Omega}{\Omega_p}\right)^{2N}\right] \quad \text{--- ②}$$

From figure at $\Omega = \Omega_p$ the attenuation is equal to $-\alpha_p$.

∴ Eqn ②, becomes,

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$$20\log\left|H(j\Omega_p)\right| = -10\log\left[1+\varepsilon^2\left(\frac{\Omega_p}{\Omega_p}\right)^{2N}\right] = -\alpha_p$$

$\varepsilon \rightarrow$ Epsilon

$$\alpha_p = 10\log\left[1+\varepsilon^2\right]$$

$$0.1\alpha_p = \log(1+\varepsilon^2)$$

$$10^{0.1\alpha_p} = (1+\varepsilon^2)$$

$$\varepsilon^2 = 10^{0.1\alpha_p}-1 \longrightarrow (3)$$

$$\varepsilon = \sqrt{10^{0.1\alpha_p}-1} \longrightarrow (3a)$$

From figure at $\Omega = \Omega_s$ the minimum stopband frequency is $-\alpha_s$.

Therefore eqn (2) becomes,

$$20\log\left|H(j\Omega_s)\right| = -\alpha_s = -10\log\left[1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right]$$

$$\alpha_s = 10\log\left[1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right]$$

$$0.1\alpha_s = \log\left[1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right]$$

Taking antilog on bothsides,

$$10^{0.1\alpha_s} = \left[1+\varepsilon^2\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}\right]$$

From eqn (3a),

$$\varepsilon^2 = 10^{0.1\alpha_p}-1$$

$$10^{0.1\alpha_s}-1 = \left(10^{0.1\alpha_p}-1\right)\left(\frac{\Omega_s}{\Omega_p}\right)^{2N}$$

$$\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1} = \left(\frac{\Omega_s}{\Omega_p}\right)^{2N}$$

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Taking log on both sides,

$$\log\left[\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right] = 2N \log\left(\frac{\Omega_s}{\Omega_p}\right).$$

$$2N = \frac{\log\left[\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right]}{\log\left[\dfrac{\Omega_s}{\Omega_p}\right]}.$$

$$N = \frac{\log\left[\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right]}{2\log\left[\dfrac{\Omega_s}{\Omega_p}\right]}$$

$$= \frac{\frac{1}{2}\log\left[\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right]}{\log\left[\dfrac{\Omega_s}{\Omega_p}\right]}$$

$$N \geq \frac{\log\left[\dfrac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}\right]^{1/2}}{\log\left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

To round off the higher integer,

i.e.,

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$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \left[\frac{\Omega_s}{\Omega_p}\right]}$$

$$N \geq \frac{\log \left[\lambda/\varepsilon\right]}{\log \left[\frac{\Omega_s}{\Omega_p}\right]}$$

where,

$$\boxed{\varepsilon = \sqrt{10^{0.1\alpha_p}-1}} \quad , \quad \boxed{\lambda = \sqrt{10^{0.1\alpha_s}-1}}$$

$$A = \lambda/\varepsilon \quad ; \quad K = \frac{\Omega_p}{\Omega_s}$$

where $K \rightarrow$ transition ratio.

N is order eqn for low pass butterworth analog filter,

$$N \geq \frac{\log A}{\log\left(\frac{1}{K}\right)}$$

Steps to design an analog butterworth low pass filter :

STEP - 1 :

From the given specifications find the order of the filter.

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log \left[\frac{\Omega_s}{\Omega_p}\right]}$$

STEP. 2 :

Round off it to next higher integer.

STEP. 3 :

Find the Transfer fn $H(s)$ for $\Omega_c = 1$ rad/sec for the value of 'N'.

Calculate the value of cut-off frequency.

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}}$$

STEP. 4 :

Find the Transfer fn $H_a(s)$ for the above value of Ω_c by substituting

$$s \rightarrow \frac{s}{\Omega_c} \text{ in } H(s)$$

ORDER N	Denominator of H(s) for $\Omega_c = 1$ rad/sec
1	$s+1$
2	$s^2 + \sqrt{2}s + 1$
3	$(s+1)\,(s^2+s+1)$
4	$(s^2+0.76537s+1)\,(s^2+1.8477s+1)$
5	$(s+1)(s^2+0.61803s+1)(s^2+1.61803s+1)$
6	$(s^2+1.931855s+1)\,(s^2+\sqrt{2}s+1)$ $(s^2+0.51764s+1)$.

For the given specification design an Analog Butterworth filter $\alpha_p = 1\,dB$, $\alpha_s = 30\,dB$. $\Omega_p = 200$ rad/sec, $\Omega_s = 600$ rad/

Sol:

STEP 1:

$$N \geq \frac{\log \sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log \sqrt{\frac{10^{(0.1\times 30)} - 1}{10^{(0.1\times 1)} - 1}}}{\log\left(\frac{600}{200}\right)}$$

999

0.9

$$N \geq \frac{\log(62.1148)}{\log 3}$$

$$N \geq \frac{1.793}{0.477}$$

$$\boxed{N \geq 3.75}$$

STEP 2:

$$N \geq 3.75$$

$$\boxed{N = 4}$$

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STEP-3 :

$$H(S) = \frac{1}{(S^2+0.7653S+1)(S^2+1.8477S+1)} \quad \text{—(1)}$$

Calculated the value for Ω_c.

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p}-1)^{1/2N}} = \frac{200}{(10^{0.1\times 1}-1)^{1/2\times 4}}$$

$$= \frac{200}{(0.258)^{1/8}} = \frac{200}{(0.258)^{0.125}}$$

$$\Omega_c = 236.90 \text{ rad/sec}$$

STEP-4 :

$$S \to \frac{S}{\Omega_c} = \frac{S}{236.90}$$

Sub $S \to \frac{S}{236.90}$ in Eqn (1).

$$Ha(S) = \frac{1}{\left(\left(\frac{S}{236.90}\right)^2 + 0.7653\left(\frac{S}{236.90}\right)+1\right)\left[\left[\frac{S}{236.90}\right]^2 + 1.8477\left[\frac{S}{236.90}\right]+1\right]}$$

$$= \frac{1}{\left[(1.78\times10^{-5}S)^2+(3.23\times10^{-3}S)+1\right]\left[[1.78\times10^{-5}S]^2+(7.79\times10^{-3}S)+1\right]}$$

$$= \frac{1}{(1.78\times10^{-5})^2\left[\left[S^2+\frac{3.23\times10^{-3}S}{1.78\times10^{-5}}+\frac{1}{1.78\times10^{-5}}\right]\left[S^2+\frac{7.79\times10^{-3}S}{1.78\times10^{-5}}+\frac{1}{1.78\times10^{-5}}\right]\right]}$$

$$3.156\times10^{7}$$

$$= \frac{}{\left[S^2+181.46S+56\times10^{-3}\right]\left[S^2+437.6S+5.617\times10^{3}\right]}$$

Thank You!