SNS COLLEGE OF ENGINEERING

(Autonomous)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

UNIT- I

Discrete Fourier Transform

Introduction to DFT

DISCRETE FOURIER TRANSFORM :

$$DFT\{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{\frac{-j2\pi kn}{N}} \; ; \; k=0,1,\ldots N-1$$

INVERSE DISCRETE FOURIER TRANSFORM :

$$IDFT\{X(K)\} = x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(K) \cdot e^{\frac{j2\pi kn}{N}} \; ;$$

$$n = 0, 1, \ldots N-1.$$

compute 4-point DFT of the sequence $x(n) = \{1, 1, 0, 0\}$.

Sol:

$\boxed{N=4}$

$x(n) = \{1, 1, 0, 0\}$

W.K.T, DFT

$$X(K) = \sum_{n=0}^{N-1} x(n) \; e^{\frac{-j2\pi kn}{N}} \; ; \; K=0,1,\ldots N-1$$

$K = 0, 1, \ldots (N-1)$

$K = 0, 1, \ldots (4-1)$

$\boxed{K = 0, 1, 2, 3}$

$$X(K) = \sum_{n=0}^{3} x(n) \; e^{\frac{-j2\pi kn}{4}}$$

$$= x(0) \; e^{\frac{-j2\pi k(0)}{4}} + x(1) \; e^{\frac{-j2\pi k(1)}{4}} + x(2) \; e^{\frac{-j2\pi k(2)}{4}} + x(3) \; e^{\frac{-j2\pi k(3)}{4}}.$$

$$= x(0)\ e^{0} + x(1)\ e^{-j\pi k/2} + x(2)\ e^{-j\pi k} + x(3)\ e^{-j\frac{3\pi k}{2}}.$$

$\because x(0)=1,\ x(1)=1,\ x(2)=0,\ x(3)=0.$

$$= 1\cdot 1 + 1\cdot e^{-j\pi k/2} + 0 + 0.$$

$$\left[\because e^{-j\theta} = \cos\theta - j\sin\theta\right].$$

$$\boxed{X(k) = 1 + \cos\left(\frac{\pi k}{2}\right) - j\sin\left(\frac{\pi k}{2}\right)}.$$

$k = 0, 1, 2, 3.$

Put $k=0$.

$$X(0) = 1 + \cos\left(\frac{0}{2}\right) - j\sin\left(\frac{0}{2}\right).$$

$$X(0) = 1 + \cos(0) - j\sin(0) = 1+1. = 2 \,//$$

$$X(1) = 1 + \cos\left(\frac{\pi}{2}\right) - j\sin\left(\pi/2\right).$$

$$= 1 + 0 - j\ \cancel{\sin}(1)$$ (calc radian mode)

$$X(1) = 1 - j \,//$$

$$X(2) = 1 + \cos\left(\frac{2\pi}{2}\right) - j\sin\left(\frac{2\pi}{2}\right).$$

$$= 1 + (-1) - j(0).$$

$$= 0. \,//$$

$$X(3) = 1 + \cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right)$$

$$= 1 + 0 - j(-1).$$

$$X(3) = 1 + j \,//$$

ANSWER:

$$X(k) = \{2, 1-j, 0, 1+j\}.$$

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Compute 4-point DFT of the following sequence

$$x(n) = \begin{cases} 1 & ; \ 0 \le n \le 2 \\ 0 & ; \ \text{otherwise} \end{cases}$$

Sol:

$$x(n) = \{1, 1, 1, 0\}$$

so, $N = 4$.

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n) \, e^{\frac{-j2\pi kn}{N}} \quad ; \quad k = 0, 1, \ldots N-1.$$

$k = 0, 1, \ldots N-1$

$= 0, 1, \ldots 4-1$

$k = 0, 1, 2, 3$

$$X(k) = \sum_{n=0}^{4-1} x(n) \, e^{\frac{-j2\pi kn}{4}}$$

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$$= \sum_{n=0}^{3} x(n)\, e^{\frac{-j2\pi kn}{4}}$$

$$= x(0)\cdot e^{\frac{-j2\pi k0}{4}} + x(1)\, e^{\frac{-j2\pi k}{4}} + x(2)\, e^{\frac{-j2\pi 2k}{4}} + x(3)\, e^{\frac{-j2\pi k3}{4}}$$

$$= x(0)\, e^{0} + x(1)\, e^{\frac{-j\pi k}{2}} + x(2)\, e^{-j\pi k} + x(3)\, e^{\frac{-j\pi k3}{2}}$$

$\because x(0) = 1,\ x(1) = 1,\ x(2) = 1,\ x(3) = 0$.

$$= 1\cdot 1 + 1\cdot e^{-j\pi k/2} + 1\cdot e^{-j\pi k} + 0 .$$

w.k.T, $\boxed{e^{-j\theta} = \cos\theta - j\sin\theta}$

$$X(k) = 1 + \cos\left(\frac{\pi k}{2}\right) - j\sin\left(\frac{\pi k}{2}\right) + \cos\pi k - j\sin\pi k .$$

Sub the values of k,

$k = 0, 1, 2, 3$

$$X(0) = 1 + \cos\left(\frac{\pi 0}{2}\right) - j\sin\left(\frac{\pi 0}{2}\right) + \cos 0 - j\sin 0 .$$

$$= 1 + 1 - 0 + 1 - 0$$

$$= 3 .$$

$$X(1) = 1 + \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) + \cos\pi - j\sin\pi$$

$$= \cancel{1} + 0 - j(1) - \cancel{1} - 0$$

$$= -j$$

$$X(2) = 1 + \cos\left(\frac{\cancel{2}\pi}{\cancel{2}}\right) - j\sin\left(\frac{\cancel{2}\pi}{\cancel{2}}\right) + \cos 2\pi - j\sin 2\pi$$

$$= \cancel{1} + \cancel{(-1)} - 0 + 1 + 0 .$$

$$= 1 .$$

$$X(3) = 1 + \cos(3\pi/2) - j\sin\left(\frac{3\pi}{2}\right) + \cos 3\pi - j\sin 3\pi$$

$$= \cancel{1} + 0 - j(-1) - \cancel{1} + 0 .$$

$$= +j$$

ANSWER :- $X(k) = \{3, -j, 1, +j\}$.

compute 8-point DFT of the following

$$x(n) = \begin{cases} 1 & ; \ 0 \le n \le 2 \\ 0 & ; \ \text{otherwise} \end{cases}$$

Sol:

$$x(n) = \{1, 1, 1, 0, 0, 0, 0, 0\}$$

$N = 8$.

DFT

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-\frac{j2\pi kn}{N}} \quad ; \ k = 0, 1, \ldots N-1$$

$k = 0, 1, \ldots \ldots 8-1$

$k = 0, 1, 2, 3, 4, 5, 6, 7.$

$$X(k) = \sum_{n=0}^{7} x(n)\, e^{-\frac{j2\pi kn}{N}}$$

$$= x(0)\, e^{0} + x(1)\, e^{-\frac{j2\pi k}{8}} + x(2)\, e^{-\frac{j2\pi 2k}{8}} + x(3)\, e^{-\frac{j2\pi 3k}{8}} + x(4)\, e^{-\frac{j2\pi 4k}{8}} + x(5)\, e^{-\frac{j2\pi 5k}{8}} + x(6)\, e^{-\frac{j2\pi 6k}{8}} + x(7)\, e^{-\frac{j2\pi 7k}{8}}$$

$$= 1.e^{0} + 1.\, e^{-\frac{j2\pi k}{8}} + 1.\, e^{-\frac{j2\pi 2k}{8}} + 0 + 0 + 0 + 0 + 0.$$

$$X(k) = 1 + e^{-\frac{j\pi k}{4}} + e^{-\frac{j\pi k}{2}}.$$

$k = 0, 1, 2, 3, 4, 5, 6, 7.$

$X(0) = 1 + e^{0}$ ...

$$\therefore \boxed{e^{-j\theta} = \cos\theta - j\sin\theta}$$

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$$X(K) = 1 + \cos\left(\frac{\pi K}{4}\right) - j\sin\left(\frac{\pi K}{4}\right) + \cos\left(\frac{\pi K}{2}\right) - j\sin\left(\frac{\pi K}{2}\right)$$

$$X(0) = 1 + \cos 0 - j\sin 0 + \cos 0 - j\sin 0.$$
$$= 1 + 1 + 0 + 1 + 0.$$
$$= 3.$$

$$X(1) = 1 + \cos\left(\frac{\pi}{4}\right) - j\sin\left(\frac{\pi}{4}\right) + \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right)$$
$$= 1 + 0.707 - j\,0.707 + 0 - j$$
$$= 1.707 - j\,1.707 = 1.707(1-j).$$

$$X(2) = 1 + \cos\left(\frac{2\pi}{\cancel{4}2}\right) - j\sin\left(\frac{2\pi}{\cancel{4}2}\right) + \cos\left(\frac{\cancel{2}\pi}{\cancel{2}}\right) - j\sin\left(\frac{2\pi}{2}\right)$$
$$= \cancel{1} + 0 - j + (\cancel{-1}) + 0.$$
$$= -j$$

	0	$\pi/2$
	0	1 s
	1	0 c

$$X(3) = 1 + \cos\left(\frac{3\pi}{4}\right) - j\sin\left(\frac{3\pi}{4}\right) + \cos\left(\frac{3\pi}{2}\right) - j\sin\left(\frac{3\pi}{2}\right).$$
$$= 1 - 0.707 - j0.707 + 0 + j.$$
$$= 0.293 + 0.293j.$$
$$= 0.293(1+j).$$

$$X(4) = 1 + \cos\left(\frac{\cancel{4}\pi}{\cancel{4}}\right) - j\sin\left(\frac{\cancel{4}\pi}{\cancel{4}}\right) + \cos\left(\frac{\overset{2}{\cancel{4}}\pi}{\cancel{2}}\right) - j\sin\left(\frac{\overset{2}{\cancel{4}}\pi}{\cancel{2}}\right)$$
$$= \cancel{1} - \cancel{1} + 0 + 1 + 0.$$
$$= 1.$$

$$X(5) = 1 + \cos\left(\frac{5\pi}{4}\right) - j\sin\left(\frac{5\pi}{4}\right) + \cos\left(\frac{5\pi}{2}\right) - j\sin\left(\frac{5\pi}{2}\right)$$
$$= 1 - 0.707 + j\,0.707 + 0 - j$$
$$= 0.293 - 0.293j$$
$$= 0.293(1-j).$$

$$X(6) = 1 + \cos\left(\frac{6\pi}{4}\right) - j\sin\left(\frac{6\pi}{4}\right) + \cos\left(\frac{\overset{3}{\cancel{6}}\pi}{\cancel{2}}\right) - j\sin\left(\frac{\overset{3}{\cancel{6}}\pi}{\cancel{2}}\right)$$

$$= \cancel{1} + 0 + j \cancel{-1} + 0$$

$$= j$$

$$X(7) = 1 + \cos\left(\frac{7\pi}{4}\right) - j\sin\left(\frac{7\pi}{4}\right) + \cos\left(\frac{7\pi}{2}\right) - j\sin\left(\frac{7\pi}{2}\right)$$

$$= 1 + 0.707 + j\,0.707 + 0 + j$$

$$= 1.707 + 1.707j$$

$$= 1.707(1+j).$$

ANSWER:

$$X(K) = \{3,\ 1.707(1-j),\ -j,\ 0.293(1+j),\ 1,\ 0.293(1-j),\ j,\ 1.707(1+j)\}.$$

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Find the IDFT of $X(K) = \{1, 0, 1, 0\}$

Sol:

IDFT:

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) \cdot e^{\frac{j2\pi kn}{N}}, \text{ where } n = 0, 1, \ldots, N-1$$

Here, $X(K) = \{1, 0, 1, 0\}$

$\boxed{N=4}$

$$x(n) = \frac{1}{4} \sum_{K=0}^{3} X(K) \; e^{\frac{j2\pi Kn}{4}}$$

$$x(n) = \frac{1}{4}\left[X(0) \cdot e^{0} + X(1)\, e^{\frac{j2\pi n}{4}} + X(2)\, e^{\frac{j2\pi 2n}{4}} + X(3)\; e^{\frac{j2\pi 3n}{4}}\right]$$

$$x(n) = \frac{1}{4}\left[X(0) \cdot e^{0} + X(1)\, e^{\frac{j\pi n}{2}} + X(2)\, e^{j\pi n} + X(3)\; e^{\frac{j\pi 3n}{2}}\right]$$

$n = 0, 1, 2, 3.$

$$x(0) = \frac{1}{4}\left[1 \cdot 1 + 0 \cdot e^{\frac{j\pi 0}{2}} + 1 \cdot e^{j\pi 0} + 0 \cdot e^{\frac{j3\pi 0}{2}}\right].$$

$$= \frac{1}{4}\,[1+1]$$

$$= \frac{2}{4}$$

$$= 2.$$

$X(0) = 1,\; X(1) = 0,\; X(2) = 1,\; X(3) = 0.$

$$= \frac{1}{4}\left[1 \cdot 1 + 0 + 1 \cdot e^{j\pi n} + 0\right].$$

$$x(n) = \frac{1}{4}\left[1 + \cos\pi n + j\sin\pi n\right].$$

$n = 0, 1, 2, 3$

$$x(0) = \frac{1}{4}\left[1 + \cos 0 + j\sin 0\right].$$

$$= \frac{1}{4}\,[1+1] = \frac{1}{\cancel{4}_2} \cdot \cancel{2} = \frac{1}{2}.$$

$$x(1) = \frac{1}{4}\left[1 + \cos\pi + j\sin\pi\right]$$

$$= \frac{1}{4}\,[1 + -1 + 0]$$

$$= 0$$

$$x(2) = \frac{1}{4}\left[1 + \cos 2\pi - j\sin 2\pi\right]$$

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$\frac{1}{4}[1+1]$

$x(2) = 1/2$.

$x(3) = \frac{1}{4}\left[1 + \cos 3\pi - j\sin 3\pi\right]$

$= \frac{1}{4}[1-1]$

$x(3) = 0$.

ANSWER:

$x(n) = \{1/2, 0, 1/2, 0\}$.

Thank You!

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