SNS COLLEGE OF ENGINEERING

(Autonomous)

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

UNIT- I

Discrete Fourier Transform

Circular Convolution

CIRCULAR CONVOLUTION :

Def:

If $x(n)$ contains L no. of samples and $h(n)$ has M no. of samples such that $L > M$, then we perform circular convolution b/w the two sequences using $N = \max(L, M)$.

By adding $(L-M)$ no. of zero sample to the sequence $h(n)$.

There are two methods,

   i) concentric circle method
   ii) Matrix method

i) concentric circle method:

PROCEDURE:

Given $x_1(n)$ and $x_2(n)$ write the values of,

   i) $x_1(n) \rightarrow$ outer circle in counter [00] anti clockwise direction.

   ii) $x_2(n) \rightarrow$ inner circle in clockwise direction

   iii) multiplying corresponding samples and sum the product to produce the output.

   iv) Rotate the inner circle one sample at a time in counter clockwise direction and Repeat step iii).

PROBLEM :-

1) Find the circular convolution of two sequences,

$x_1(m) = \{1, 2, 2, 1\}$; $x_2(n) = \{1, 2, 3, 1\}$

using concentric circle method & Matrix method.

Sol:

i) concentric circle method: $\boxed{P = ?}$

$x_1(n) = \{1, 2, 2, 1\} \Rightarrow L = 4$

$x_2(n) = \{1, 2, 3, 1\} \Rightarrow M = 4$.

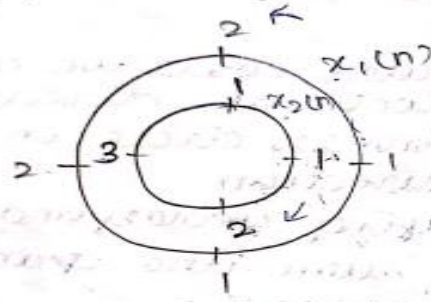For circular convolution,

   $y(n) = x_1(n) \, Ⓝ \, x_2(n)$.

∴ length of the output $y(n) \rightarrow N$

$$N = \max(L, M)$$
$$N = \max(4, 4)$$

$$\boxed{N = 4}$$
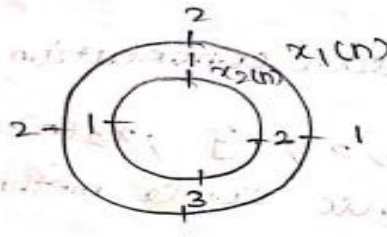
STEP.1 :

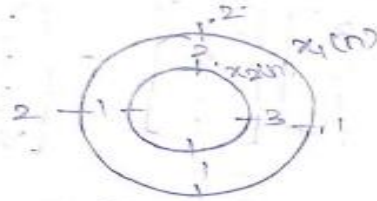$$y(0) = 1 + 2 + 6 + 2$$

$$\boxed{y(0) = 11}$$

STEP. 2 :

$$y(1) = 2 + 3 + 2 + 2$$
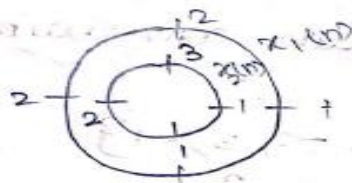
$$\boxed{y(1) = 9}$$

STEP·3:

$$y(2) = 1 + 3 + 4 + 2$$

$$\boxed{y(2) = 10}$$

STEP·4:

$$y(3) = 1 + 1 + 6 + 4$$

$$\boxed{y(3) = 12}$$

RESULT:

$$y(n) = \{11, 9, 10, 12\}$$
$$(\because N = 4)$$

ii) Matrix method:

$$\begin{bmatrix} x_2(0) & x_2(3) & x_2(2) & x_2(1) \\ x_2(1) & x_2(0) & x_2(3) & x_2(2) \\ x_2(2) & x_2(1) & x_2(0) & x_2(3) \\ x_2(3) & x_2(2) & x_2(1) & x_2(0) \end{bmatrix} \begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$$x_1(n) = \{1, 2, 2, 1\}$$
$$x_2(n) = \{1, 2, 3, 1\}$$

$$\begin{bmatrix} 1 & 1 & 3 & 2 \\ 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

$$y(n) = \begin{bmatrix} 1+2+6+2 \\ 2+2+2+3 \\ 3+4+2+1 \\ 1+6+4+1 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 10 \\ 12 \end{bmatrix}$$

Find the circular convolution of two sequences,

$$x_1(n) = \{1, -1, -2, 3, -1\}$$
$$x_2(n) = \{1, 2, 3\}$$

using concentric circle method & Matrix method.

sol: $y(n) = \{8, -2, -1, -4, -1\}$.

$$x_1(n) = \{1, -1, -2, 3, -1\} \Rightarrow L = 5$$
$$x_2(n) = \{1, 2, 3\} \qquad \Rightarrow M = 3$$

$$\Rightarrow (L-m)$$
$$= (5-3) = 2 \text{ zeros}.$$
$$x_2(n) = \{1, 2, 3, 0, 0\}$$

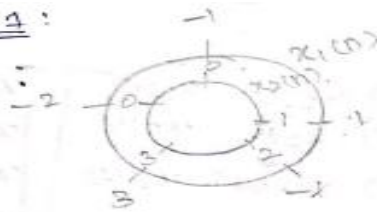For circular convolution,
$$y(n) = x_1(n) \ⓝ\ x_2(n).$$

$\therefore$ length of the o/p $y(n) \rightarrow N$.
$$N = \max(L, M)$$
$$N = \max(5, 3)$$
$$\boxed{N = 5}$$
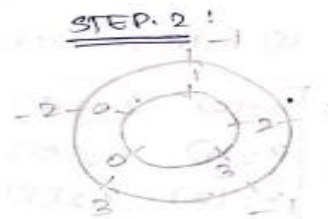
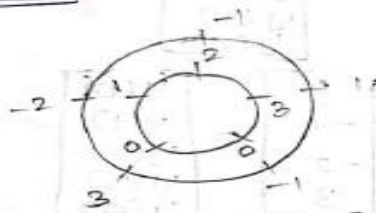STEP. 1 :

$y(0) = 0 + 0 + 9 - 2 + 1$

$\boxed{y(0) = 8}$

STEP. 2 :

$y(1) = 0 + 0 - 1 - 3 + 2$

$\boxed{y(1) = -2}$
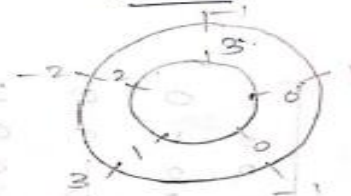
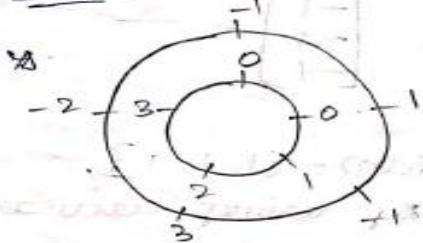STEP. 3 :

$y(2) = 0 + 0 + 3 - 2 - 2$

$\boxed{y(2) = -1}$

STEP. 4 :

$y(3) = 3 + 0 + 0 - 3 - 4$

$\boxed{y(3) = -4}$

STEP. 5 :

$y(4) = 6 - 1 + 0 + 0 - 6$

$\boxed{y(4) = -1}$

RESULT :

$y(n) = \{ 8, -2, -1, -4, -1 \}$

$\boxed{\therefore N = 5}$

Thank You!