



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Topic: 1.6 – t-Distribution -Difference of mean

Student's 't' Test for difference of mean.

Sample size n_1, n_2
mean \bar{x}_1, \bar{x}_2

Test Statistics:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$
$$S^2 = \frac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}$$

Note:- If the samples are given
then $\bar{x}_1 = \frac{\sum x_1}{n_1}$ $\bar{x}_2 = \frac{\sum x_2}{n_2}$
$$S^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

Degrees of freedom = $n_1 + n_2 - 2$

Problems 1:

1) The average number of articles produced by two machines per day are 200 and 250 with S.D 20 and 25 respectively on the basis of records of 25 days production. Can you regard both the machines equally efficient at 1% level of significance.



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Soln: $n_1 = 25$ $n_2 = 25$
 $\bar{x}_1 = 200$ $\bar{x}_2 = 250$
 $s_1 = 20$ $s_2 = 25$

$$s^2 = \frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2} = \frac{25(20)^2 + 25(25)^2}{25 + 25 - 2}$$
$$s^2 = 533.85$$
$$s = 23.10$$

$H_0: \mu_1 = \mu_2$
 $H_1: \mu_1 \neq \mu_2$

L.O.S: 1%

Dof: $25 + 25 - 2 = 48$

Test Statistic

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$
$$= \frac{200 - 250}{23.10 \sqrt{\frac{1}{25} + \frac{1}{25}}} = -7.65$$

$|t| = 7.65$



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

Critical value .
 $\alpha = 1\%$ Dof 28 $\Rightarrow t_2 = 2.32$.
C.V T.V
7.65 \geq 2.32
 H_0 rejected.

2). Below are given the gain in weights (in lbs) of pigs fed on the diets A and B

Diet A	25	32	30	34	24	14	32	24	30
Diet B	44	34	22	10	47	31	40	30	32

31 35 25 - -
35 18 21 35 29 22

Test if the two diets differ significantly as regards their effect on increase in weight

Soln:-

$n_1 = 12, \quad n_2 = 15$

$\bar{x}_1 = \frac{336}{12} = 28 \quad \bar{x}_2 = \frac{450}{15} = 30$



SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

$$\begin{aligned} \sum (x_1 - \bar{x}_1)^2 &= 380 \\ \sum (x_2 - \bar{x}_2)^2 &= 1410 \\ S^2 &= \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2} \\ &= \frac{380 + 1410}{12 + 15 - 2} = 71.6 \end{aligned}$$

\bar{x}_1	$x_1 - \bar{x}_1$	$(x_1 - \bar{x}_1)^2$	x_2	$(x_2 - \bar{x}_2)^2$	$(x_2 - \bar{x}_2)$
25	-3	9	44	14	196
32	4	16	34	4	16
30	2	4	22	-8	64
34	6	36	10	-20	400
21	-4	16	47	17	289
14	-14	196	31	1	1
32	4	16	40	10	100
24	-4	16	30	0	0
30	2	4	32	2	4
31	3	9	35	5	25
35	7	49	18	-12	144
25	-3	9	21	-9	81
			35	5	25
			29	-1	1
<u>336</u>		<u>380</u>	<u>22</u>	<u>-8</u>	<u>64</u>
			450		1410



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

$H_0: \mu_1 = \mu_2$
 $H_1: \mu_1 \neq \mu_2$
Level of Significance: 5%
Dof: $12 + 15 - 2 = 25$

29) Test Statistics -
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{28 - 30}{\sqrt{71.6 \left(\frac{1}{12} + \frac{1}{15} \right)}} = -0.609$$

 $|t| = 0.609$
Critical Value: $t_{\alpha/2} = 2.06$ at 25
 $t_{\alpha} = 2.06$
Conclusion: C.V T.V
 $0.609 < 2.06$
 H_0 accepted.

3). Two horses A and B were tested according to the time (in seconds) to run a particular track with the following results

Horse A:	28	30	32	33	33	29	34
Horse B:	29	30	30	24	27	27	-

$\bar{x}_1 = \frac{\sum x_1}{n} = \frac{219}{7} = 31.3$ $\bar{x}_2 = \frac{\sum x_2}{n} = \frac{174}{6} = 29$



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

x_1	$x_1 - \bar{x}_1$	$(x_1 - \bar{x}_1)^2$	x_2	$(x_2 - \bar{x}_2)$	$(x_2 - \bar{x}_2)^2$
28	-3.3	10.89	29	1.2	1.44
30	-1.3	1.69	30	2.2	4.84
32	0.7	0.49	30	2.2	4.84
33	1.7	2.89	24	-3.8	14.44
33	1.7	2.89	27	-0.8	0.64
29	-2.3	5.29	27	-0.8	0.64
34	2.7	7.29			
		<u>31.43</u>			<u>26.84</u>

$$s^2 = \frac{\sum (x_1 - \bar{x}_1)^2 + \sum (x_2 - \bar{x}_2)^2}{n_1 + n_2 - 2} = \frac{31.43 + 26.84}{7 + 6 - 2}$$
$$s^2 = 5.29$$
$$s = 2.3$$

$H_0: \mu_1 = \mu_2$
 $H_1: \mu_1 \neq \mu_2$
L.O.S: 5%
Dof: $n_1 + n_2 - 2 = 7 + 6 - 2 = 11$
Test Statistics
$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{31.3 - 27.8}{2.3 \sqrt{\frac{1}{7} + \frac{1}{6}}} = 2.73$$



SNS COLLEGE OF ENGINEERING

Kurumbapalayam (Po), Coimbatore – 641 107

AN AUTONOMOUS INSTITUTION



Approved by AICTE, New Delhi & Affiliated to Anna University, Chennai

3) $|t| = 2.73$

Critical value: $\alpha = 5\%$ Dof 11

$t_{\alpha} = 2.2$

Conclusion: C.V T.V

$2.73 > 2.2$

H_0 rejected.