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Topic: 1.5 – t-distribution – Single mean

Test of Significance of Small
Sample size < 30
Tests based on small samples
i) Student's 't' test
ii) F-test
iii) χ^2 -test (Chi-Square)

Type I
Student's 't' test.
Test statistics.
$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} \approx \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

 $\bar{x} \Rightarrow$ Sample mean
 $\mu \Rightarrow$ Population mean.
 $n =$ Sample size.
 $s =$ Standard deviation.
Conclusion: e.v T.V
 $|t| < |t_{\alpha}| \quad H_0 \text{ accepted.}$
 $|t| > |t_{\alpha}| \quad H_0 \text{ rejected.}$



Note:- (17)

If S.D is not given directly, (ie)
if samples are given then (x_1, x_2, \dots)

Mean $\bar{x} = \frac{\sum x}{n}$

$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

Problems.

1. The mean weekly sales of soap bars in departmental stores was 146.3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153.7 and showed a S.D of 17.2. Was the advertising campaign successful.

Soln : $\mu = 146.3$ $n = 22$
 $\bar{x} = 153.7$ $s = 17.2$

$H_0 : \mu = 146.3$
 $H_1 : \mu > 146.3$



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L.O.S : $\alpha = 5\%$

Test Statistic

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n-1}}}$$
$$= \frac{153.7 - 146.3}{\frac{17.2}{\sqrt{22-1}}} = 1.97$$

$|t| = 1.97$

Critical value :-
 α at 5% degrees of freedom 21.
 $t_{\alpha} = 1.721$

Conclusion .
C.V T.V
 $1.97 > 1.721$
 $\therefore H_0$ rejected .



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2). The mean life time of a sample of 25 fluorescent light bulbs produced by a company is computed to be 1570 hours with S.D of 120 hours. The company claims that the average life of the bulbs produced by the company is 1600 hours. Using the level of significance of 0.05. Is the claim acceptable.

$n = 25$ $\bar{x} = 1570$ $\sigma = 120$
 $\mu = 1600$
Degree of freedom = $25 - 1 = 24$
 $H_0: \mu = 1600$
 $H_1: \mu \neq 1600$
L.O.S: $\alpha = 5\%$
Degrees of freedom = $n - 1 = 25 - 1 = 24$.



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$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{1570 - 1600}{120/\sqrt{24}}$$
$$= \frac{-30}{24.49} = -1.22$$
$$t = -1.22 \quad |t| = 1.22$$

Critical value

$$\alpha = 5\% \quad \text{Def} = 24$$
$$t_{\alpha} = 2.064$$

Conclusion :

C.V	T.V
1.22	2.064

$1.22 < 2.064$

H_0 accepted.

3). A random sample of 10 boys had the following I.Q's. 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean IQ of 100? find



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Find the reasonable range in which most of the mean I.Q values of samples of 10 boys lie.

$$\bar{x} = \frac{\sum x}{n} = \frac{972}{10} = 97.2$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
70	-27.2	739.84
120	22.8	519.84
110	12.8	163.84
101	3.8	14.44
88	-9.2	84.64
83	-14.2	201.64
95	-2.2	4.84
98	0.8	0.64
107	9.8	96.04
100	2.8	7.84
<u>972</u>		<u>1833.60</u>

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1833.60}{9} = 203.73$$
$$S.D = \sqrt{\text{Variance}} = \sqrt{203.73} = 14.27$$

$H_0: \mu = 100$
 $H_1: \mu \neq 100$
L.O.S = 5%



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Test Statistic

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{97.2 - 100}{14.27/\sqrt{10}}$$
$$= -0.62$$

$|t| = 0.62$

Critical value

$\alpha = 5\%$ at $n = 9$. $t_{\alpha} = 2.262$

Conclusion:

C.V	T.V
0.62	2.262

$\therefore H_0$ accepted

4). The following table gives the lengths of 12 students samples of Egyptian cotton taken from a large consignment 48, 46, 49, 46, 52, 45, 43, 47, 47, 46, 47, so Test if the mean length of the consignment be taken as 46



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$$\bar{x} = \frac{\sum x}{n} = \frac{566}{12} = 47.16$$

x	$x - \bar{x}$	$(x - \bar{x})^2$
48	0.84	0.71
46	-1.16	1.35
49	1.84	3.39
46	-1.16	1.35
52	4.84	23.43
45	-2.16	4.67
43	-4.16	17.31
47	-0.16	0.03
47	-0.16	0.03
46	-1.16	1.35
47	-0.16	0.03
50	2.84	8.07
<u>566</u>		<u>61.72</u>

Level of significance : 5%
Test Statistics



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$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$
$$= \frac{47.16 - 46}{2.37/\sqrt{12}} = 1.69$$

$t = 1.69.$

Critical value

$\alpha = 5\%$ Dof = $12 - 1 = 11$

$t_{\alpha} = 2.20$

Conclusion :-

C.V	T.V
1.69	> 2.20

H_0 accepted.